Frozen in Flux Theorem

$$\frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{v} \times \vec{B})}_{\text{Induction Portion}} + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}}_{\text{Diffusion Portion}}$$

→ Induction equation governing magnetic field diffusion in solid / liquid (gas, fluid) → plasma.

Induction Portion

$E = \vec{v} \times \vec{B}$ (Lorentz force)

Curl of E

~~is mag~~

→ like Maxwell Equations.

Assume conductivity is uniform throughout fluid/Solid.

Diffusion Portion → lost to heat.

How to get this big Guy?

Ohm's Law : $\vec{J} = \sigma(\vec{E} + (\vec{v} \times \vec{B}))$

Plug in Maxwell Equations ~~III~~ IV → Ampère Maxwell

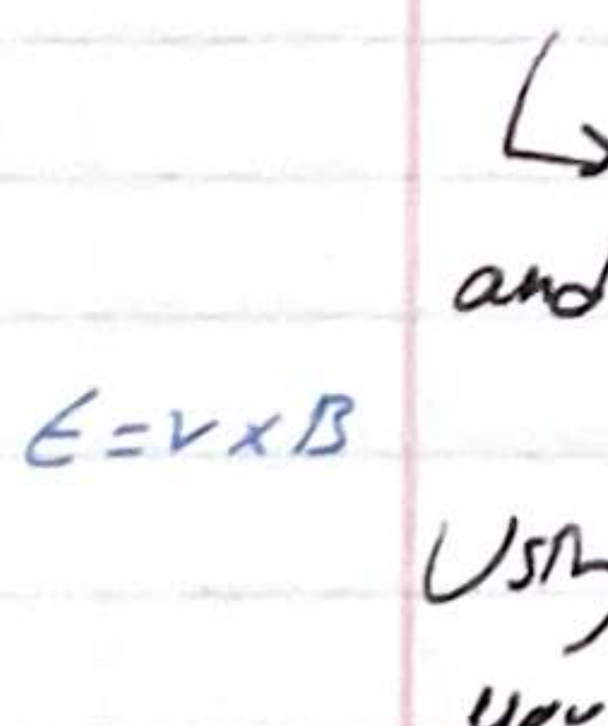
Then Plug that in Maxwell Equation III → Faraday's law

↳ Get rid of Cross products using Levi-civita Symbol and convert to Dot (delta kronecker)

$E = v \times B$

→ ~~Gauss's~~ No magnetic Monopole.

Using $\nabla \cdot \vec{B} = 0$ (Maxwell II) & Vector Calculus, you get the equation above.



Dropping Some terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times B) + \cancel{\frac{1}{\mu_0 \sigma}} \nabla^2 \vec{B}$$

Dropped if $R_m \gg 1$

When Conductivity tends to ∞, diffusive terms drop.
Molten Core, Sun, Plasma

Effect → Magnetic fields cannot diffuse out & remain frozen in the fluid.

When Conductivity $R_m \ll 1$, Diffusion of magnetic field is like heat equation.

$\cancel{\nabla^2 f = 0}$

Real valued

$$\boxed{\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial t^2}} \rightarrow \sim \text{Laplace Equation}$$

↓ vector valued + in 3D

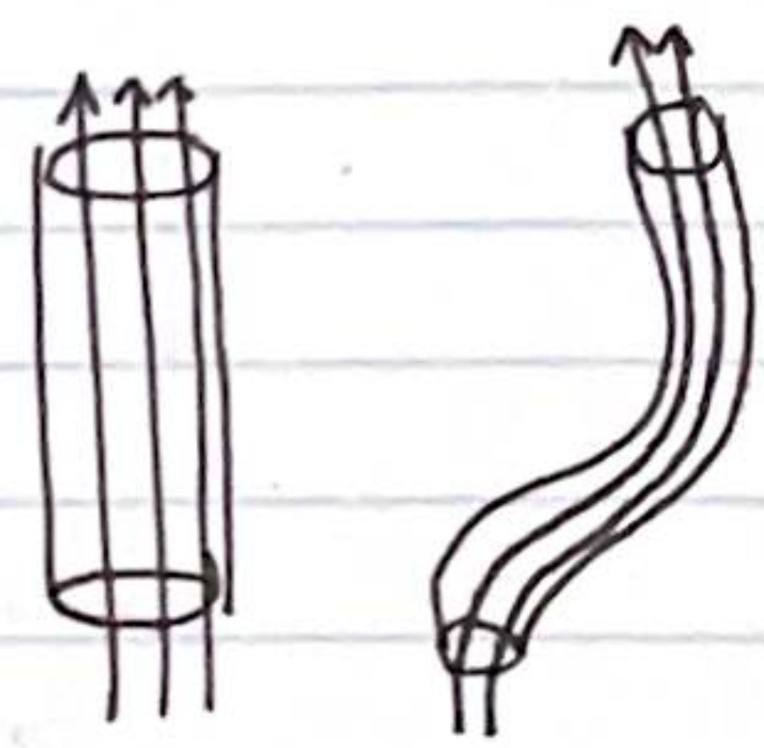
$$\boxed{\frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}} \rightarrow \text{Like Laplace Equation.}$$

Magnetic Reconnection Producing Solar Flares

is same as Magnetotail reconnection accelerating electrons back at poles.

Violation of ~~the~~ Alfven's Theorem

↳ States that Magnetic flux is frozen in fluid



This occurs because despite Sun being $R_m >> 1$ denoting absence of diffusive term, there are regions where R_m is small enough to allow diffusion to dominate.

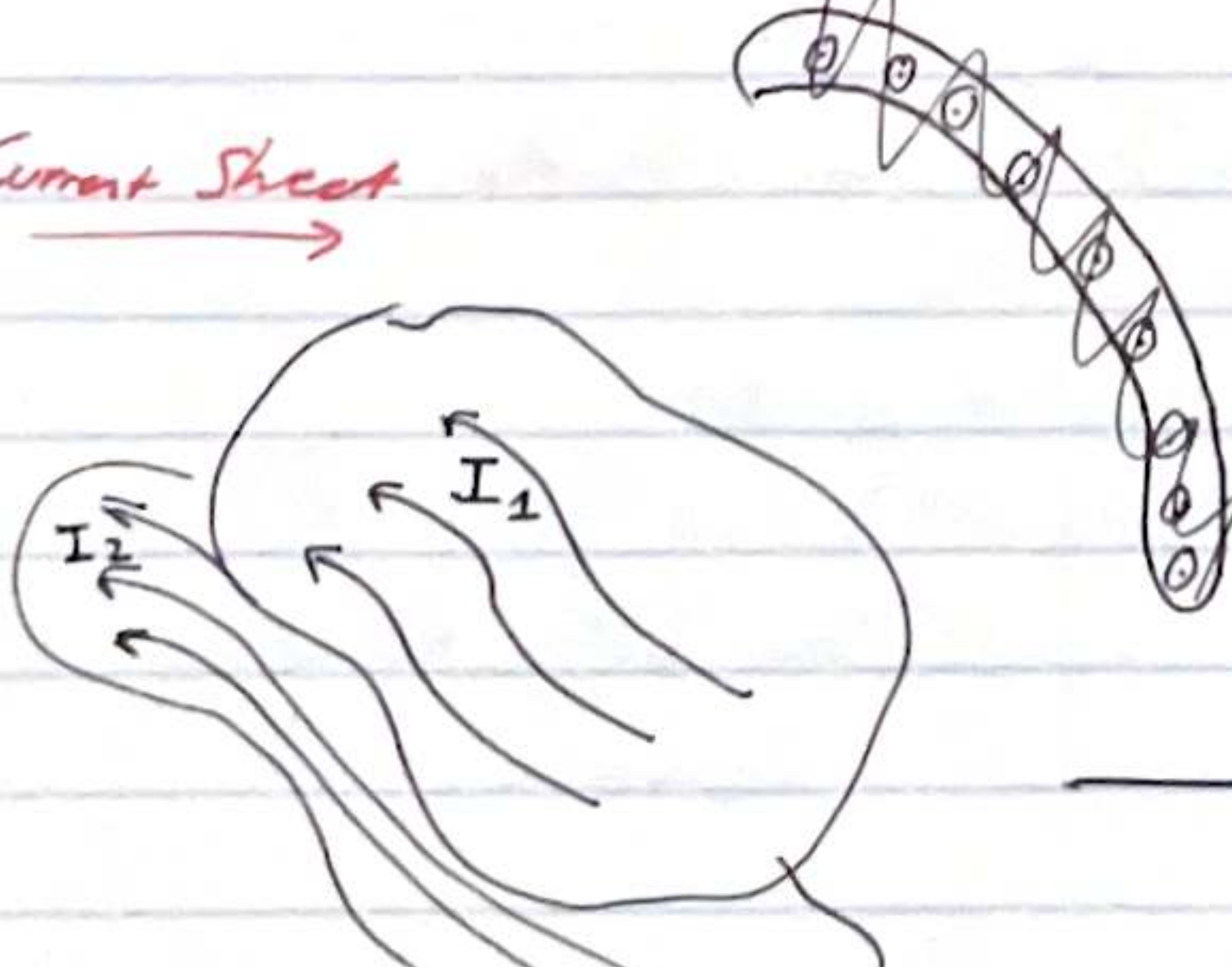
↳ fields move from high field region to low field region.

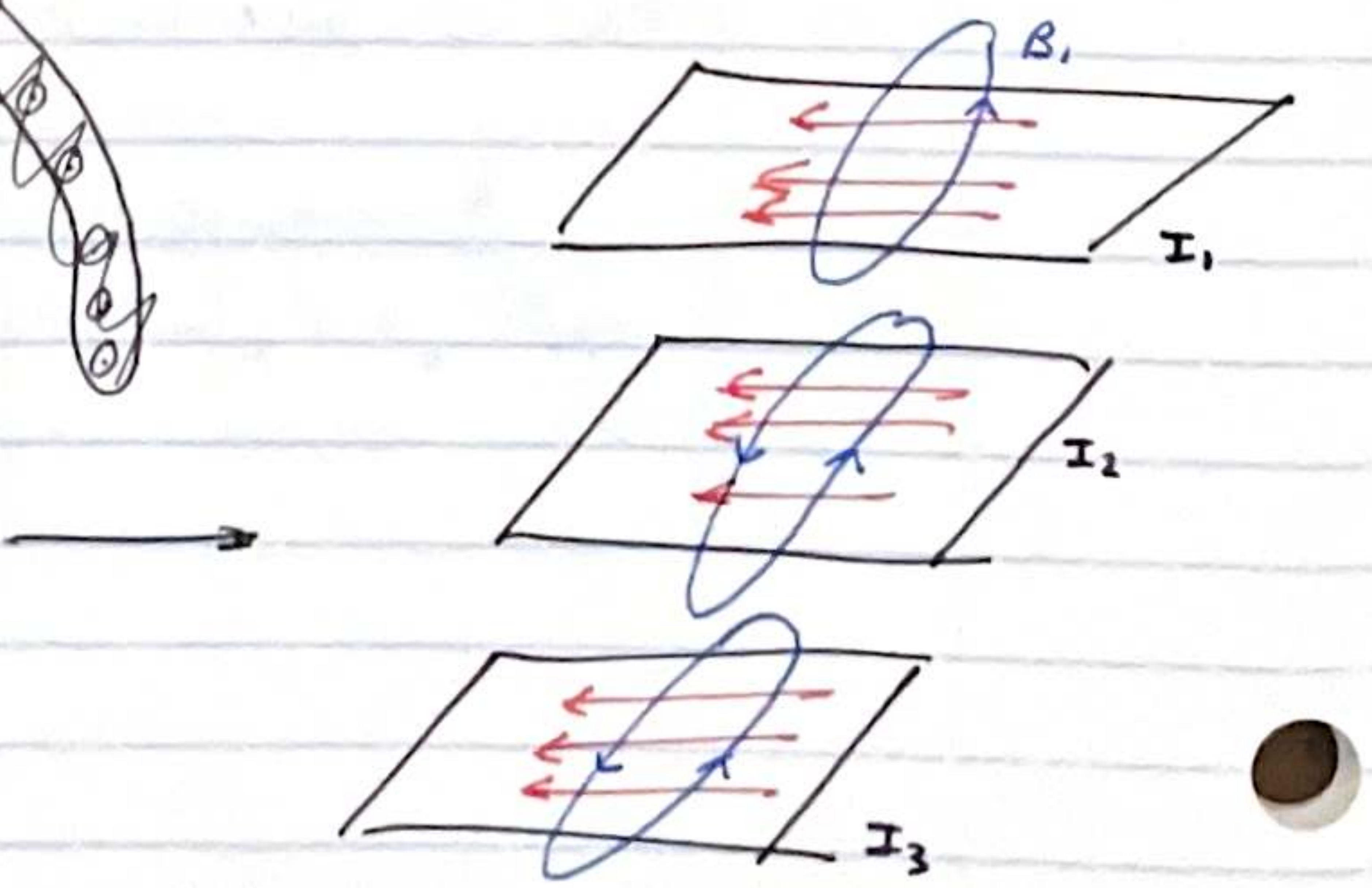
What happens in these regions? $\nabla \times \vec{B} = \mu J + \cancel{\mu\epsilon \frac{\partial E}{\partial t}} \to 0$

- μJ: Current from Plasma flow.
- $\frac{\partial E}{\partial t}$: Current from time varying Electric field. → Small in Plasma.

$\boxed{\text{Mechanism of diffusion is magnetic reconnection?}}$

Current Sheet





Frozen flux undergo differential rotation to & causes tension to build up.

This tension is released when diffusive term dominates & the mechanism is magnetic reconnection.

How does magnetic tension occur?

~~[illegible]~~

Ohm's law : $\vec{J} = \sigma\left(\vec{E} + (\vec{v} \times \vec{B})\right)$

Ampère Maxwell law : $\nabla \times \vec{B} = \mu_0 \vec{J} + \underbrace{\cancel{\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}}_{0}$

Displacement Current is negligible in Plasma

Plug Ohm's law in Ampère-Maxwell

↳ $\nabla \times \vec{B} = \mu_0 \sigma\left(\vec{E} + (\vec{v} \times \vec{B})\right)$

$$\vec{E} + (\vec{v} \times \vec{B}) = \frac{1}{\mu_0 \sigma} \nabla \times \vec{B}$$

Faraday Law : $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{E} = \frac{1}{\mu_0 \sigma}(\nabla \times \vec{B}) - (\vec{v} \times \vec{B})$$

$$\nabla \times \vec{E} = \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B}) - \nabla \times (\vec{v} \times \vec{B})$$

↳ To get rid of that thing, Use Levi-Civita identity & its relation to delta kronecker

$$\begin{aligned} \nabla \times (\nabla \times \vec{A}) &= \varepsilon_{ijk} \varepsilon_{kmn} \partial_m A_n \\ &= \varepsilon_{kij} \varepsilon_{kmn} \partial_j \partial_m A_n \\ &= \partial_j \partial_i A_j - \partial_j \partial_j A_i \\ &= \partial_i \partial_j \partial_j - \partial_j \partial_j A_i \\ &= \nabla(\nabla \cdot A) - \nabla^2 A \end{aligned}$$

$$\Rightarrow -\frac{\partial \vec{B}}{\partial t} = -\frac{1}{\mu_0 \sigma} \nabla^2 B + - \nabla(\vec{v} \times \vec{B})$$

$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}$$

$$\Rightarrow \nabla \times \vec{E} = \frac{1}{\mu_0 \sigma}\left(\nabla \underbrace{\cancel{(\nabla \cdot B)}}_{0} - \nabla^2 B\right)$$

No magnetic Monopoles
Gauss's Law

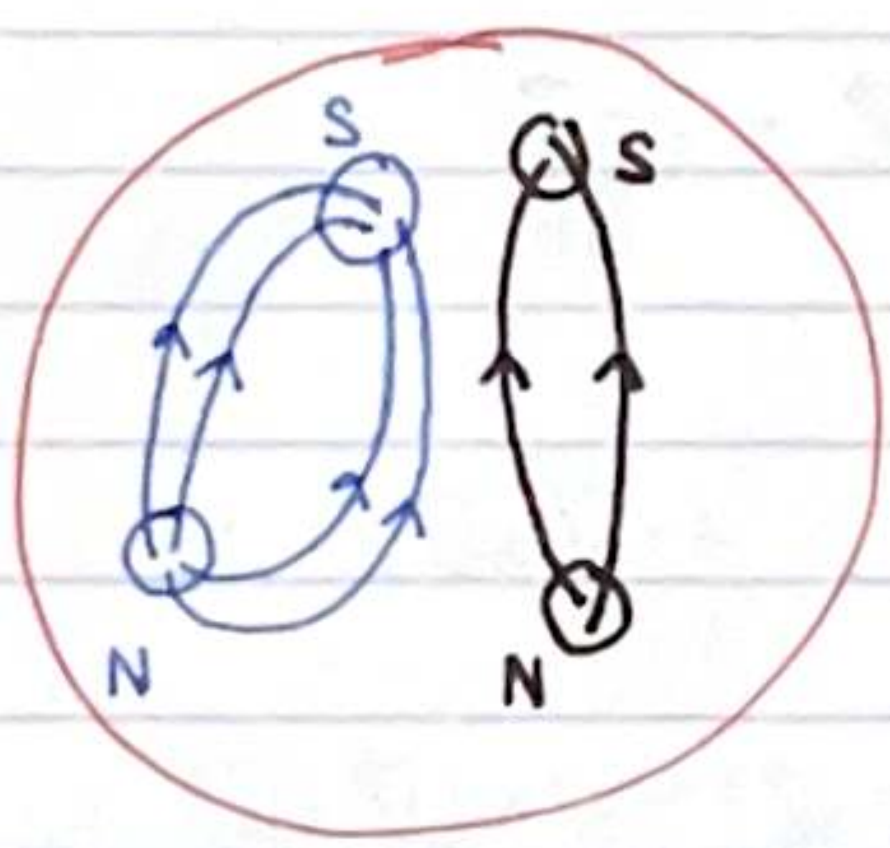
Deviations from

Ideal MHD & Alfven's Theorem

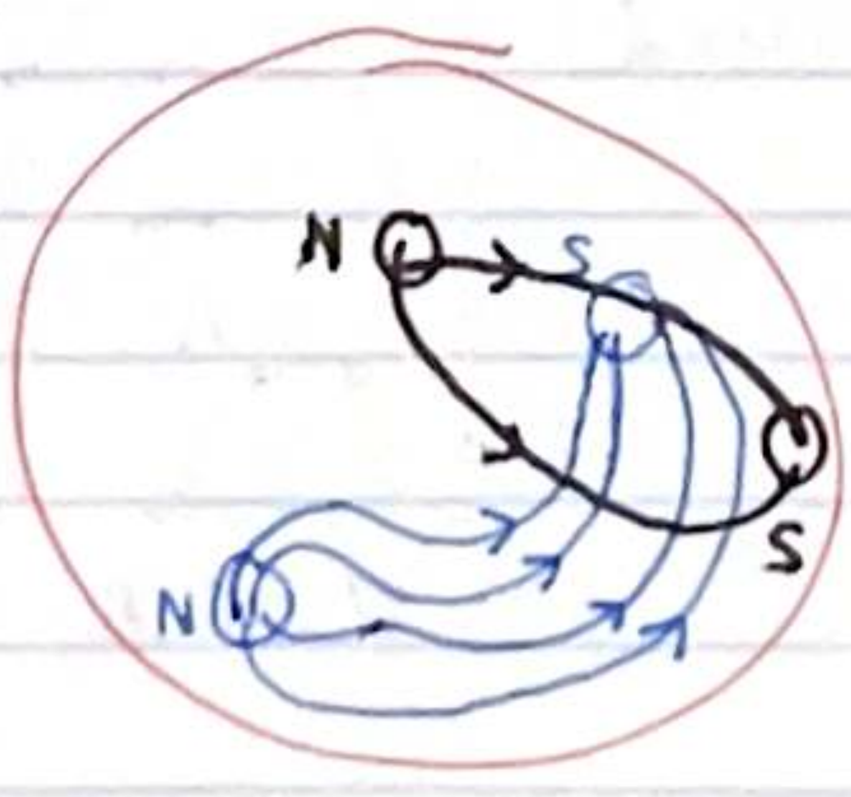
Frozen if flux theorem
↳ Plasma is tied to the magnetic field lines.
  ↳ It cannot diffuse out & thus remains locked & moves with the plasma.

However, the sun despite being large & very conductive, does not always have ideal MHD conditions.

If Sun was ideal, magnetic domains with their plasma would have preserved topology irrespective of transformations.



Deform fluid.



Preservation of Domains.

In Certain regions like Sun Spots, there are intense magnetic activities.

These regions have increased resistivity. Magnetic field concentrate through space & time.

↓

Turbulence & Current sheets

↳ Thin regions with sudden change in magnetic field direction over short distances.

In ideal MHD, to release the tension of the magnetic field lines would require bulk motion of the plasma over a long time.

↓

No rapid & localized energy release.

↳ Same stuff happen magnetotail. Reconnection.

How Plasma Cause front of Geomagnet to open up.

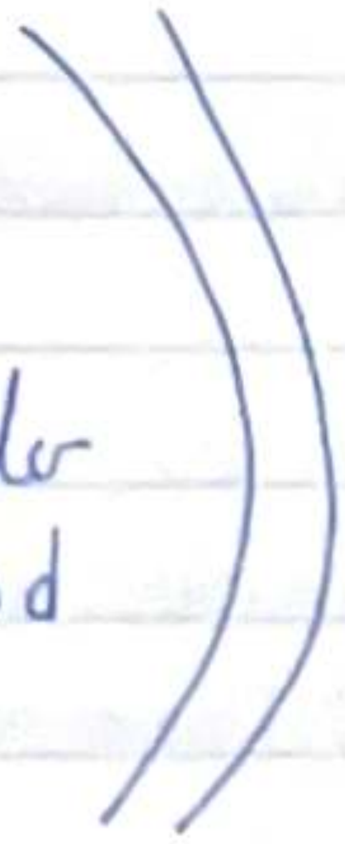
Reconnection would not be possible w/o magnetic diffusion.

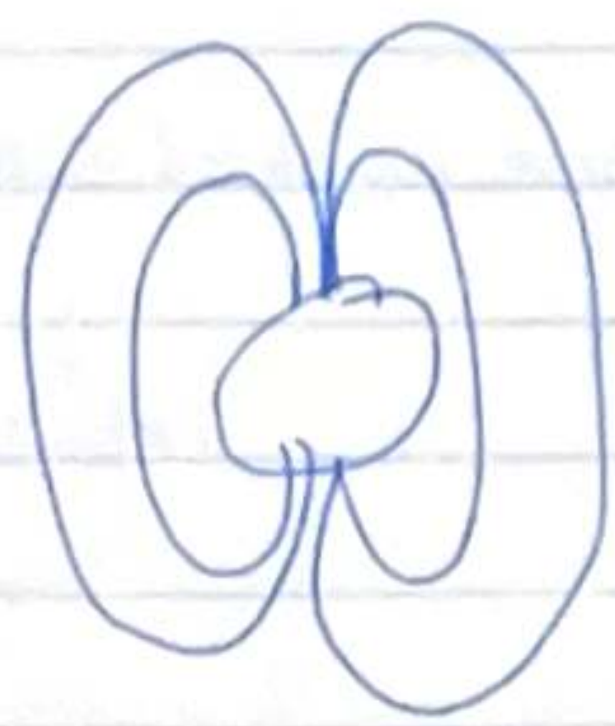
↳ Cannot change topology which is essential for reconnection.

Differential Topology

Separatrix

Solar Wind CME





$$\sigma(\vec{E} + \vec{v} \times B) = \vec{J}$$

$$\nabla \times B = \mu_0(\sigma(E + \vec{v} \times \vec{B}))$$

$$\nabla \times B = \mu_0 \sigma(E + \vec{v} \times \vec{B})$$

$$\nabla \times B = \mu_0 \sigma$$

$$-\frac{\partial \vec{B}}{\partial t} =$$

$$\frac{1}{\mu_0 \sigma} \nabla \times \vec{B} = \vec{E} + \vec{v} \times \vec{B}$$

$$\vec{E} = \nabla \times \vec{B} - (\vec{V} \times \vec{B})$$

$$\nabla \times \vec{E} = \frac{1}{\mu_0 \sigma} \nabla \times \nabla \times \vec{B} - \nabla(\vec{V} \times \vec{B})$$