

## Hertzsprung-Russell diagram (H-R diagram)

\* Stellar classification system - Different types.

In 1911, Danish astronomer Ejnar Hertzsprung plotted the absolute magnitude of stars against their colour (effective temperature). Independently, in 1913 the American astronomer Henry Norris Russell used Spectral class against absolute magnitude. Their resultant plot showed that the relationship between temperature and luminosity of a star was not random but appeared to fall into distinct groups.

$$m - M = 5 \log \left( \frac{d}{10} \right)$$

$m$ : apparent magnitude of star  
 $M$ : absolute magnitude of star  
 $d$ : distance between star and Earth

\* Apparent magnitude ( $m$ ) - Number that measures brightness of a celestial object as seen from Earth.

## Absolute Magnitude

$$M_{\text{Sun}} = -26.7$$

brightest object in the sky.

Absolute magnitude is a measure of the luminosity of a celestial object on a logarithmic astronomical magnitude scale. An object's absolute magnitude is defined to be equal to the apparent magnitude that the object would have if it were viewed from a distance of 10 parsecs (32.6 light years) with no extinction or dimming of its light due to absorption by interstellar dust particles. By hypothetically placing all objects at a standard reference distance from the observer, their luminosities can be directly compared on a magnitude scale. The more luminous an object, the smaller the numerical value of its absolute magnitude. A difference of 5 magnitudes between the absolute magnitude of 2 objects corresponds to a ratio of 100 in their luminosities.

A difference of  $n$  magnitudes in absolute magnitude corresponds to a luminosity ratio of  $(100)^{\frac{n}{5}}$

Absolute magnitude can be specified for different wavelength ranges corresponding to specific ~~bands or~~ filters or passbands; for stars, a commonly quoted absolute magnitude is the absolute visual magnitude which uses the visual (V) band of the spectrum in the UBV photometric system. Absolute magnitudes are denoted by  $M$  with subscript representing the filter band used for measurement eg  $M_V$  for absolute magnitude in the V band.

An object's absolute bolometric magnitude represents its total luminosity over all wavelength, rather than in a single filter band, as expressed on a logarithmic magnitude scale. To convert from absolute magnitude in a specific filter band to a bolometric magnitude, a bolometric correction is applied.



$$BC_K = M_{\text{bol}} - M_K$$

$M_K$ : Absolute magnitude value

$BC_K$ : bolometric correction value in K band

For solar system bodies that shine in reflected light, a different definition of absolute magnitude ( $H$ ) is used based on a standard reference distance of 1 astronomical unit.



## Effective Temperature

The effective temperature of a body (such as star or planet) is the temperature of a black body that would emit the same total amount of electromagnetic radiation.

When the star or planet's net emissivity in the relevant wavelength band is less than unity, (less than that of a black body) the actual temperature of the body will be higher than the effective temperature. The net emissivity may be low due to surface or atmospheric properties, including green house effect.

Effective temperature is often used as an estimate of a body's surface temperature when the body's emissivity curve (as a function of wavelength) is not known.

The effective temperature of a star is the temperature of a black body with the same luminosity per surface area ( $F_{bol}$ ) as the star and is defined according to the Stefan-Boltzmann

law,  $F_{bol} = \sigma T_{eff}^4$

## Stefan - Boltzmann law

The total energy radiated per unit surface area of a black body across all wavelength per unit time,  $j^*$  (also known as black body radiant emittance) is directly proportional to the fourth power of the black body's thermodynamic temperature

### Colour temperature - ~~ideal~~

It is the temperature of an ideal black body radiator that radiates light of a comparable colour to that of the light source.

> 5000K - cooler colours

(2700-3000)K - warm colours

↳ Analogy to incandescence

lighting rather than temperature, \*

$$j^* = \sigma T^4 \rightarrow j^* \propto T^4$$

Stefan-Boltzmann  
Constant

$$\sigma = \frac{2\pi^5 K^4}{15 C^2 h^3} = 5.670373 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

↳ Emit significant infrared radiation - temperature.

Radiance

$$\rightarrow L = \frac{j}{\pi} = \frac{\sigma}{\pi} T^4$$

(Watts per square metre

per steradian)

↳ solid angle

Note:  $L = 4\pi R^2 \sigma T^4$

A body that does not absorb all incident radiation (grey body) emits less total energy than a black body and is characterised by an emissivity,  $\epsilon < 1$

$$j^* = \epsilon \sigma T^4$$

↳ emissivity of grey body - Perfect black body,  $\epsilon = 1$

General  $\epsilon$  depends on wavelength (function of wavelength)  $\epsilon = \epsilon(\lambda)$

$j^*$  - radiant emittance - Has dimensions of energy flux (energy per time per unit area) - SI units ( $\text{J s}^{-1} \text{m}^{-2}$ ) or ( $\text{W m}^{-2}$ )

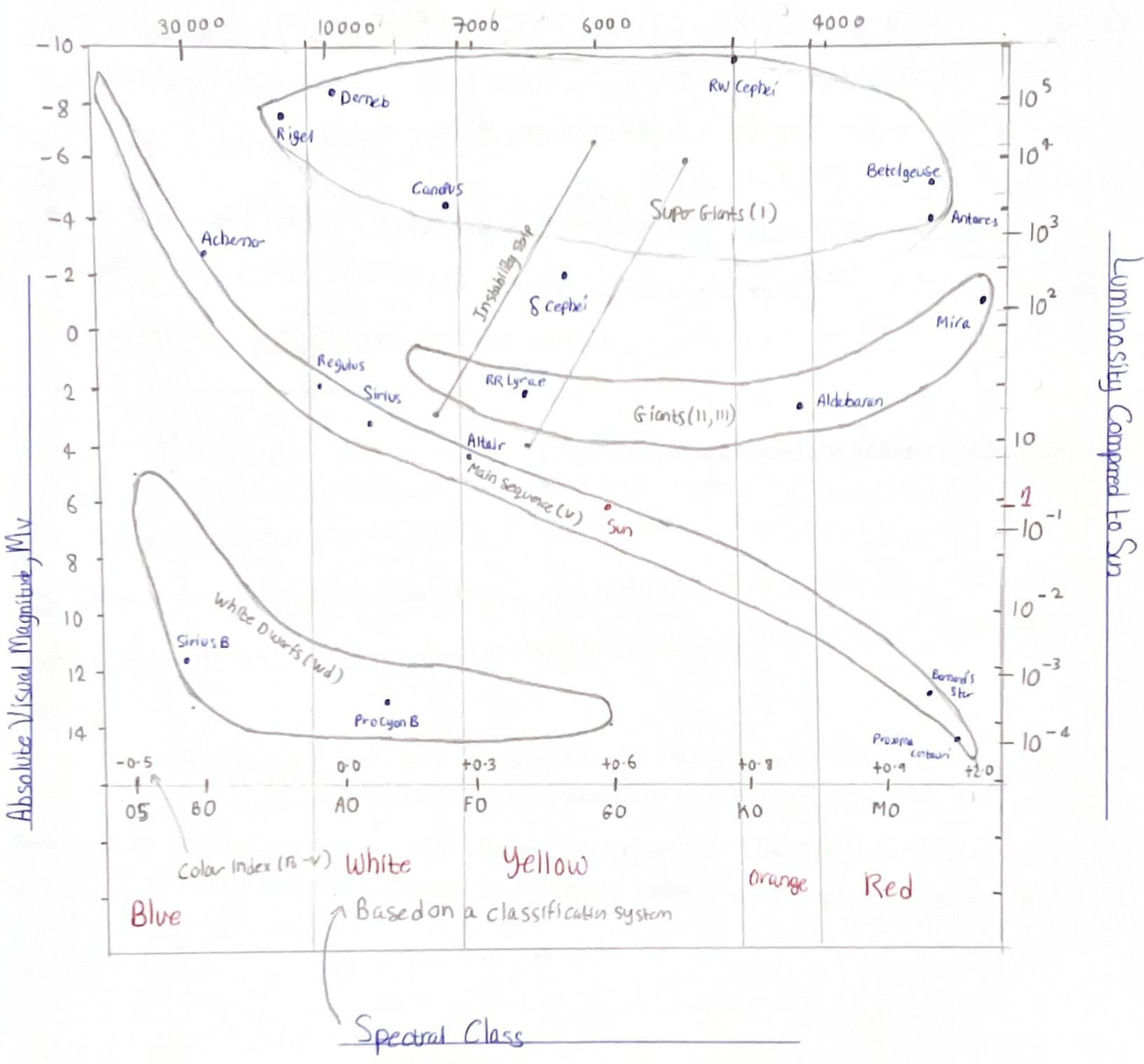
Total power radiated =  $j^* \times A$   
Surface area

$$P = A j^* = A \epsilon \sigma T^4 \rightarrow P \propto T^4$$



$q_s \rightarrow 160$   
 $n \rightarrow \frac{160}{25} (n)$

# Effective Temperature, K

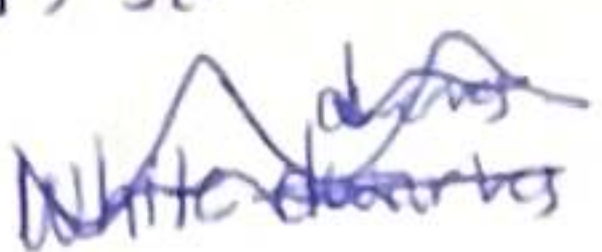


The majority of stars including Sol are found along the main sequence. These stars vary widely in effective temperature but the hotter they are, the more luminous they are. These stars are fusing hydrogen into helium in their core. Stars spend the bulk of their existence as main sequence stars



## Interpreting H-R diagram

The Super giants Rigel and Deneb have the same effective temperature as Sirius but have extremely high luminosities. They have larger radii than Sirius hence greater surface area and higher luminosities. Sirius is a main sequence star but is hotter than the red main sequence Barnard's star; it is much more luminous than Barnard's star.

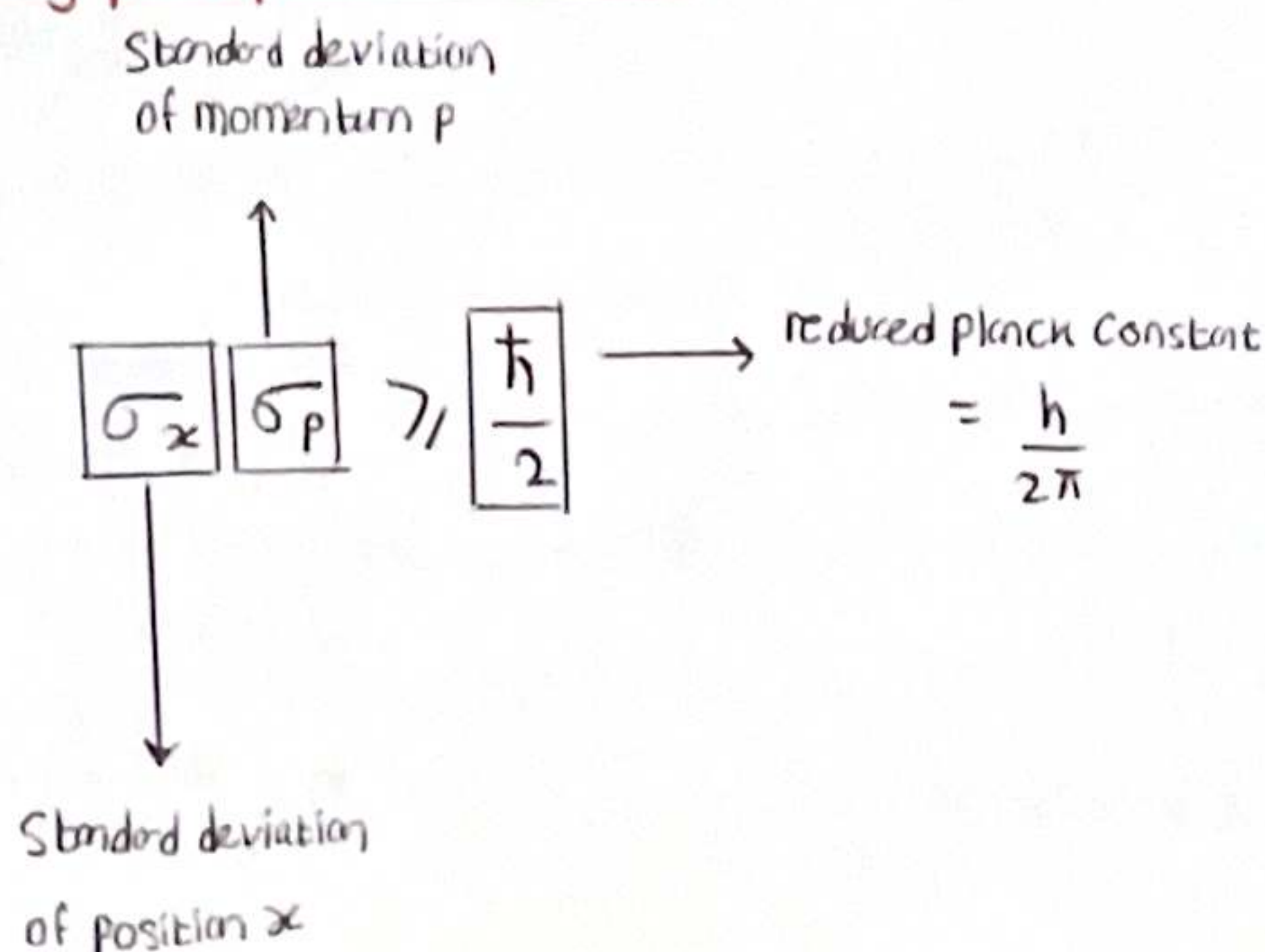
Consider White dwarf Procyon B. ~~White dwarfs~~  White dwarfs are very hot (about 10000 K hotter), therefore emit a lot of energy per second for each square metre of their surface. The fact that they are so dim however means they are extremely small and have very small surface area.

## Principle of the Sextant & Navigation techniques

{ Antenna  
ratio



# Heisenberg's Uncertainty principle.



Navier-Stokes equation  
Bernoulli equation  
Stress Tensor  
degeneracy Pressure  
uncertainty principle  
fourier transform  
Hilbert space  
manifold  
shape of hyperbolic  
trig. functions.

In quantum mechanics, the uncertainty principle is any of a variety of mathematical inequalities asserting a fundamental limit to the precision with which certain pairs of physical properties of a particle known as complementary variables such as position  $x$  and momentum  $p$  can be known.

**Note:** Not to be confused with the observer effect which notes that measurements of certain systems cannot be made without affecting the systems, that is, without changing something in the system.

The uncertainty principle is inherent in the properties of all wave-like systems, and that it arises in quantum mechanics simply due to the matter wave nature of quantum objects. Thus, the uncertainty principle actually states a fundamental property of quantum systems and is not about the observational success of current technology. It also implies that the position and momentum cannot be accurately measured at the same time.

$$\Delta x \Delta p \geq \frac{h}{2}$$

Put ①  $\Delta x = 4$   $\Delta p = 2$

eg.

$$\Delta x \Delta p \geq h$$

$$4 \times 2 = 8$$

↓ for more precise accurate position  $\Delta x$  falls

$$2 \times 4 = 8$$

↓

$$0.5 \times 16 = 8$$

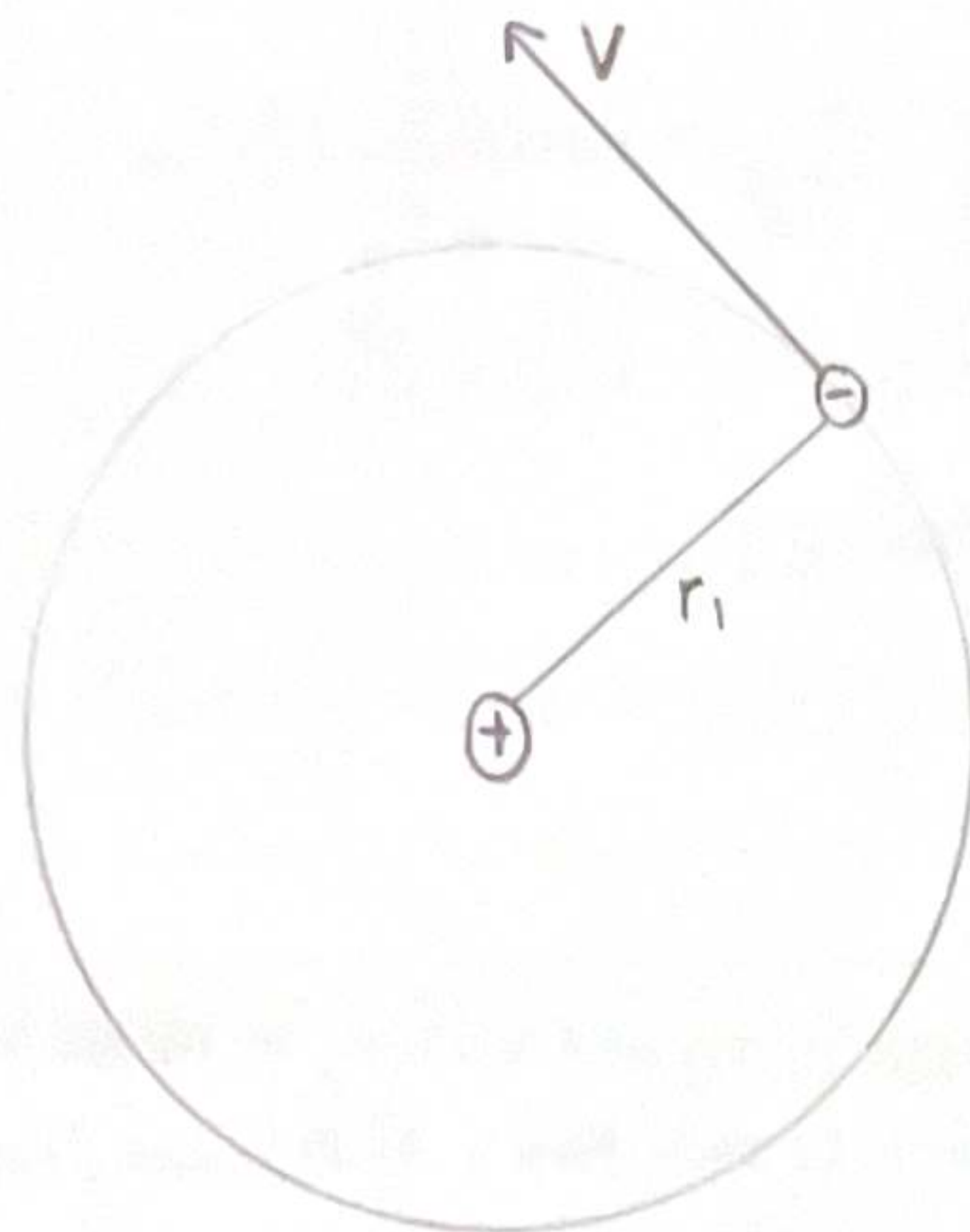
↳ When  $\Delta x$  falls,  $\Delta p$  must increase.

$$\Delta x \propto \frac{1}{\Delta p}$$

As position of a particle is more 'localised' i.e. its position is known with greater accuracy,  $\Delta x$  becomes smaller while  $\Delta p$  increases



## Bohr Atomic Model



$2r_1 = 1.06 \times 10^{-10} \text{ m} \approx \text{diameter} \approx \text{size of atom.}$

Known Values

$$r_1 = 5.3 \times 10^{-11} \text{ m}$$

$$V = 2.2 \times 10^6 \text{ m/s}$$

$$m = 9.11 \times 10^{-31} \text{ Kg}$$

$$p = mv$$

Suppose there is  $\overset{0.1}{10\%}$  uncertainty associated with velocity

$$\frac{\Delta p}{p} = \underbrace{\frac{\Delta m}{m}}_{\text{Suppose} = 0} + \frac{\Delta v}{v}$$

Suppose  
= 0

$$\% \text{ unc. in } p \text{ is } \frac{\Delta p}{p} \times 100 = \overset{0.1}{\frac{\Delta v}{v}} \times 100$$

$$\Rightarrow \Delta p = 0.1 p$$

$$= 0.1 mv$$

$$= (0.1)(9.11 \times 10^{-31})(2.2 \times 10^6)$$

$$= 2.0 \times 10^{-25} \text{ Kg m/s}$$

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

$$\Delta x \geq \frac{\hbar}{2\Delta p} \Rightarrow \Delta x \geq 2.6 \times 10^{-10} \text{ m}$$

$\Delta x$  is larger than the diameter using Bohr model  
unc. in pos<sub>n</sub> of electron

Since velocity was known  $\Rightarrow \downarrow \Delta p \Rightarrow \text{large unc. in pos}_n \text{ of } e^-$



## Electron degeneracy pressure

Electron degeneracy pressure is a particular manifestation of the more general phenomenon of quantum degeneracy pressure.

The Pauli exclusion principle disallows two identical half-integer spin particles (electrons and all other fermions) from simultaneously occupying the same quantum state. The result is an emergent pressure against compression of matter into smaller volumes of space.

Electron degeneracy pressure results from the same underlying mechanism that defines the electron orbital structure of elemental matter.

For bulk matter with no net electric charge, the attraction between electrons and nuclei exceeds (at any scale) the mutual repulsion of electrons plus the mutual repulsion of nuclei.

So, absent of electron degeneracy pressure, the matter would collapse into a single nucleus.

Freeman Dyson showed that solid matter is stabilised by quantum degeneracy pressure rather than electrostatic repulsion. Because of this, electron degeneracy pressure creates a barrier to gravitational collapse of dying stars (formation of blackholes) and is responsible for the formation of white dwarfs.



Electron degeneracy pressure will halt the gravitational collapse of a star if its mass is below the

Chandrasekhar limit (1.44 solar masses). This

is the pressure that prevents a white dwarf from collapsing. A star exceeding this limit

and without significant thermally generated pressure

will continue to collapse to form either a neutron star or

blackhole, because the degeneracy pressure provided by the

electrons is weaker than the inward pull of gravity.

Set of numbers.

Conic sections (Area ...)  
Cauchy - Stress tensor  
Fourier Transform

Hawking radiation mechanism

Planets. / Stars - properties/mechanism.

Non-relativistic.



From the Uncertainty principle.

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

A material subjected to ever increasing pressure will compact more and for electrons within it, their delocalisation will decrease ( $\Delta x$ ) ↓. Thus, as dictated by the uncertainty principle, the spread in the momentum of the electrons will increase. Thus, no matter how low temperature drops,  $e^-$ s must be travelling at the large 'Heisenberg speeds' contributing to the pressure. When the pressure due to these 'Heisenberg motions' exceeds that of the pressure from the thermal motions of the electrons, the  $e^-$ s are referred to as degenerate and the material is termed degenerate matter. This analysis is true at even almost zero temperatures.

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From Fermi gas theory.