Category Theory

A category is formed by 2 things : 1) Object(s) of the Category
2) Morphisms which relate 2 objects.
↳ Behaves similar to function composition.
↳ Associativity & Existence of id morphism.

This whole thing is a Category. (c)

$$\begin{tikzcd}
X \arrow[loop above, "1_x"] \arrow[r, "f"] \arrow[dr, "g \circ f"'] & Y \arrow[loop above, "1_y"] \arrow[d, "g"] \\
 & Z \arrow[loop right, "1_z"]
\end{tikzcd}$$

f & g are morphisms between distinct objects X, Y & Z.

1_x is the identity morphism.

- The class of objects → $ob(c)$
  (Collection w/o Russel Paradox)

- The collection of morphisms → $hom(c)$
  $f : a \mapsto b$ | f is a morphism from object a to b.
  $hom_c(a, b)$ denotes the class of all morphisms from a to b.

- Binary Operation $\circ$ st.
  $\circ : hom(b,c) \times hom(a,b) \mapsto hom(a,c)$
  $f : b \mapsto c$, $g : a \mapsto b$
  $g \circ f$ → Apply f then g

Axioms
Associativity : $h \circ (g \circ f) = (h \circ g) \circ f$
Identity :
$\forall x, \exists 1_x : x \mapsto x$
*
for every morphism $f : a \mapsto b$
$1_b \circ f = f = \cancel{1_a} f \circ 1_a$

$1_x : x \mapsto x$
Will map all x in c to itself...

1_a or 1_b composed with f yields f
The identity morphism is unique & universal to the class...

What happens if I have ~~an~~ arrow $X \cancel{\to} Y$,

What happens if $f: a \mapsto b$ and $g: c \mapsto d$

and $1_x: x \mapsto x$

$\Rightarrow 1_a : a \mapsto a$

$1_c : c \mapsto c$

I compose $1_a \circ g$?

Although the identity function will send the argument to itself, It is not in the correct domain.

For Every object X, $\exists$ $id_X : X \to X$. This is the identity morphism on X.

For every morphism $f: A \to B$, we have $\boxed{id_B \circ f} = f = \boxed{f \circ id_A}$!!!

$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, "f"] & B \arrow[loop right, "id_B"] \end{tikzcd}$$

Be careful here

Do f then id_B Sends us from A to B then from B to B

Doing id_A Sends us from A to A then A to B.

↳ In both cases terminal destination is B

Questions: Can we ~~apply~~ compose the identity morphism from say $hom(a,b)$ to ~~objects~~ ~~containing which do not belong to~~ another class $hom(c,d)$?

How is the identity morphism defined?

For Every object X in C, there exists one morphism 1_X Sending the object X to itself.

When composing morphisms between object A & B of Category C,

$$\begin{tikzcd} A \arrow[loop left, "1_A"] \arrow[r, "f"] & B \arrow[loop right, "1_B"] \end{tikzcd} \quad \boxed{1_B \circ f = f \circ 1_A} \to \text{Role of } 1_x.$$

Can I compose the identity morphism of object A to itself with object B?

Morphisms are composed with Morphisms, NOT objects...

→ A Morphism is an Arrow between 2 objects

An identity morphism is an arrow to itself.

$\forall_x \; 1_x : X \to X$

1_A is the identity morphism for A [This function sends A to itself]

Review Later

Domain, Codomain, Range, Image & Pre-Image

$f: \mathbb{R} \to \mathbb{R}$ $\quad f: x \mapsto x^2$ $\quad f(x) = x^2$

$f: Dom \to Codom$

Domain is largest possible set of inputs to $f(x)$, So it is $\mathbb{R}$

The Codomain is the Set which contains the range (So it is $\mathbb{R}$ too.)

The range is the set of all possible outputs $[0, \infty)$

~~Suppose $A \subseteq \mathbb{R}$ then $Im(A) = f(A)$~~. Suppose we have a subset S of the Domain $\{1, 2, 3\}$

f maps this subset onto the codomain

→ The Co domain is $\mathbb{R}$

The range is $[0, \infty)$

The image of S under f is $\{1, 4, 9\}$

Anyways,

$1_X : X \to X$ $\qquad\qquad\qquad\qquad$ $f : X \to Y$

The domain of 1_X is X

$f \circ 1_X = f = 1_Y \circ f$

The domains must match as required else it is not a function.

Else this function is not defined

Examples of Category.

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Splitting action of intellect.

↳ $\boxed{\text{Equivalence relation induce partition on a Set.}}$

↳ Depending on how we partition stuff

Form Categories & ~~Patt~~ maps abstracting everything else (Category Theory)

↳ Like shining light on one part of the object sending the opposite to shadow...

That's the nature of intellect & consciousness. Truth is the paradoxical tension of both sides held at the same time.

Also, more info means increasing number of bits to more precisely state something. But with increasing no of bits/microstate, more chance for it to go wrong.

↳ Entropy counts that

Ideal vs non-ideal.

· Why are scientific/Mathematical "Truths" easier?

What is the action of intellect mathematically?

– Some function acting on certain algebraic object to return certain information on that object.

– Also need to show that the function induces partition on the object a can be composed...

→ Need to show $\forall x$ in the object, f_i has an opposite, sending the inverse element of x to a shadow...

→ The inverse of the structure...

Take a Simple ~~alg~~ mathematical Structure : A collection of object with no defined rules...

↳ As general as possible. → Why not anything else?
A group, A ring, A field, An ideal,
A module? A vector Space?
Topology, metric,

A Set and its Coset...

S   Coset

f

Show that it will induce a coset!

Or Show $f : S \to Coset(s)$ (Proven in video)

fulfils the requirements of intellect...

Show that the Coset will have "Shadow" inverse element...

Video : Opposite of a Set

Vector   covector → A covector of V is a vector in the dual Space V^V

algebra   coalgebra → A coalgebra of A is an algebra of the opposite category A^{op}.

An algebra is a Set A, a collection of operations and a set of axioms that the operations must Satisfy.

$G(\mathbb{Z}, +)$ → Binary Operation (Addition) $(\mathbb{Z}, +, \times)$ ↘ Ring

↑ Set of Integers

$\exists$ Id , Associative

$\exists$ INV , Closure

Minimal Structure needed to define an algebra.

- The Algebra is an object A in the category C.
- There is a multiplication map on A, $\mu : A \times A \to A$
- There is an element in A called identity, e
  In categorical terms, ~~ther~~ $\eta : 1 \to A$ where 1 is the terminal object in the category

Reversing arrows lead to coAlgebras.