Constructive And Destructive Interference.

Two waves (with the same amplitude, frequency and wave length)

The Classical Wave equation. (1D)

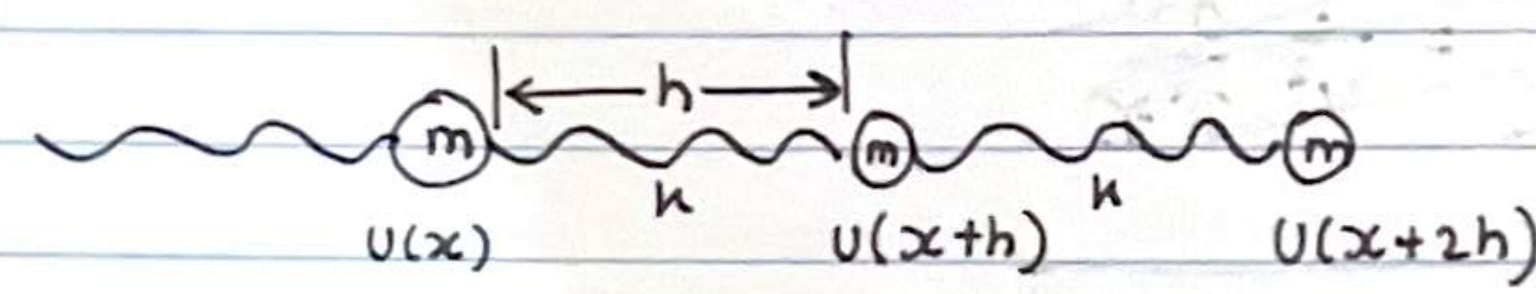
$$\cancel{\frac{\partial^2 u}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial x^2}}$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial x^2}}$$

F at $(x+h)$
$= F$ at $(x+2h)$
$-$ F at x.

Derivation of 1D classical wave equation
[from Hooke's law]

Imagine an array of little masses m interconnected with a massless springs of length h. The springs have spring constant k



↗ Time dependent.

$U(x)$ measures the distance from the equilibrium of the mass situated at x.
So, $U(x)$ measures the disturbance (strain) that is travelling in an elastic material.
The forces exerted on the mass at location $(x+h)$ are :

$F=ma \leftarrow F_{Newton} = m \cdot a(t) = m \cdot \frac{\partial^2}{\partial t^2} U(x+h,t)$

$F=ke \leftarrow F_{Hooke} = F_{x+2h} - F_x = k\left[U(x+2h,t) - U(x+h,t)\right]$

$- k\left[U(x+h,t) - U(x,t)\right]$

at $(x+h)$

Equation of motion for the weight at $(x+h)$ can be found by equating these 2.

$$\frac{\partial^2}{\partial t^2} U(x+h,t) = \frac{k}{m}\left[U(x+2h,t) - U(x+h,t) - U(x+h,t) + U(x,t)\right]$$

If the array of weights consists of N weights spaced evenly over the length $L = Nh$ of total mass $M = Nm$ and the total spring constant of the array $K = k/N$ we can rewrite the equation as :

$$\frac{\partial^2}{\partial t^2} u(x+h,t) = \frac{KL^2}{Mh^2} \left[U(x+2h,t) - 2U(x+h,t) + U(x,t)\right]$$

Taking the limits $N \to \infty$, $h \to 0$ and assuming smoothness, we get;

$$\boxed{\frac{\partial^2 u(x,t)}{\partial t^2} = \underbrace{\frac{KL^2}{M}}_{\text{Square of the propagation speed.}} \frac{\partial^2 u(x,t)}{\partial x^2}}$$

from defn of 2nd derivative

$N^2 = L^2/h^2$ $\quad \frac{k}{m}$ $\quad K = k/N$ $\quad L = Nh$

$\frac{KN^2}{M}$ $\quad \frac{KN}{M/N}$ $\quad k = KN$

$\frac{KL^2}{Mh^2}$ $\quad m = \frac{M}{N}$

~~For Sp2~~

(4) $$\nabla \times B = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$ (3) $$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\frac{\partial}{\partial t}(\nabla \times B) = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$ $\longrightarrow$ Taking time derivative of (4)

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial B}{\partial t}$$ $\longrightarrow$ Taking Curl of (3)

$$\frac{\partial}{\partial t} \nabla \times B = \nabla \times \frac{\partial B}{\partial t}$$ $\longrightarrow$ Spacial & Time derivative commute

↳ commutative property

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

So, $$\nabla(\nabla \times E) = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

From Identity : $$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) \overset{-\nabla^2 E}{} = \nabla^2 E$$

$$\Rightarrow \nabla^2 E = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

Compare to wave eqn $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

$$\frac{1}{v^2} = \varepsilon_0 \mu_0$$

$$\boxed{V = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c}$$