Alternative Method

From graph, it can be seen that every even term is zero.

$$b_n = -\frac{5}{n\pi}(\cos n\pi - 1)$$

$$= -\frac{5}{n\pi}\left((-1)^n - 1\right)$$

$$\Rightarrow \cancel{b_n} \quad \frac{10}{n\pi} \text{ if } n \text{ is odd } \& \ 0 \text{ if } n \text{ is even.}$$

to generate odd numbers for series, $b_n = \dfrac{10}{(2n-1)\pi}$, $n = 1, 2, 3, \dots$

Odd terms are also required for the Sine terms, since the even ones will be zero.

$$f(t) = \frac{5}{2} + \sum_{n=1}^{\infty} \cos\frac{n\pi t}{4} + \sum_{n=1}^{\infty} \frac{10}{(2n-1)\pi} \sin\frac{(2n-1)\pi t}{4}$$

$$= 2.5 + \frac{10}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin\frac{(2n-1)\pi t}{4}$$

$$\boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt}$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\sin\theta$, $\cos\theta$; and by euler's identity $e^{i\theta}$ form a complete orthonormal basis in L^2.

=

Any function can be expressed as a linear combination of $\sin\theta$, $\cos\theta$.

The complex plane, $e^{i\theta}$ encodes this information.

The function; periodic is presented as a rotation in $\mathbb{C}$.

However, the function must be $\in L^2$, normed & has inner product

$\downarrow$

$\boxed{\text{Hilbert Space}}$

L^2

Basis Sets fully generate the span of a vector space.

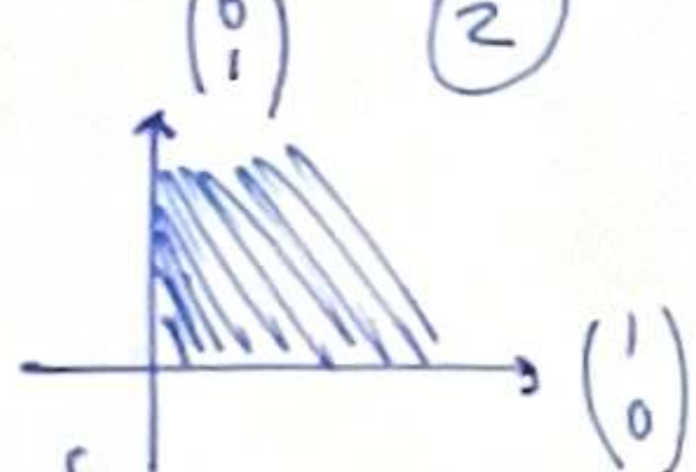
The Elements need be *linearly independent*.

(1)

- Linearly Independent
- Span $\mathbb{R}^2$
- Not Orthogonal.
- Can define inner product $\langle \cdot x, \cdot y \rangle$

$\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$

(2)



- Linearly independent
- Span $\mathbb{R}^2$
- orthogonal
  - Easier to calculate inner product.

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

(3) New concept

Eigenvectors, vectors that are only scaled by a matrix Transformation.

$\boxed{A\vec{v} = \lambda\vec{v}}$

- There is a matrix which is a ~~...~~ L·T
- There is the span of that matrix/L·T

Basis define a vector space.

A matrix is a transformation that manipulates that space.

The matrix is the ~~language~~ commands that tell the space how to move.

Either can adopt to each other.

The basis is the language of the vector space.

I just Said we use exponentials because they can capture what we need; growth, death, oscillations...

That the bone Providing the Structure of the thing. } There is growth, death, & the living in life "objectively".

What does $\boxed{e^{-st}}$ represent philosophically / Phenomenologically?

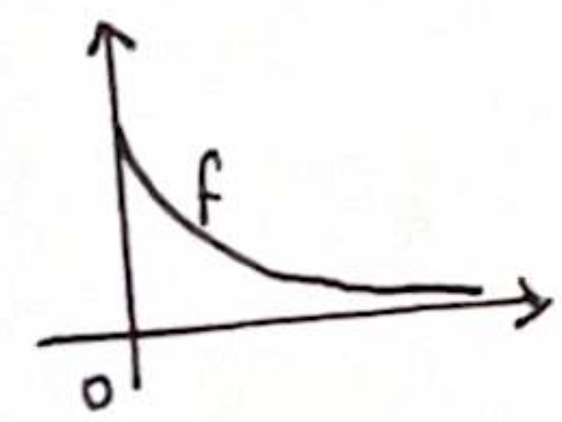
Let's look at statistics; exponential distributions, normal distribution.

$$\mathcal{L}\{f\}(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

↳ Inner Product $\boxed{\langle f(t) | e^{-st} \rangle}$

or Expected Value of $f(t)$ under exponential distribution.

$$Pr(x) = \lambda e^{-\lambda x} \qquad x \geq 0$$



$$E(f(x)) = \int_0^\infty f(t)\, \underbrace{\lambda e^{-\lambda x}}\, dx$$

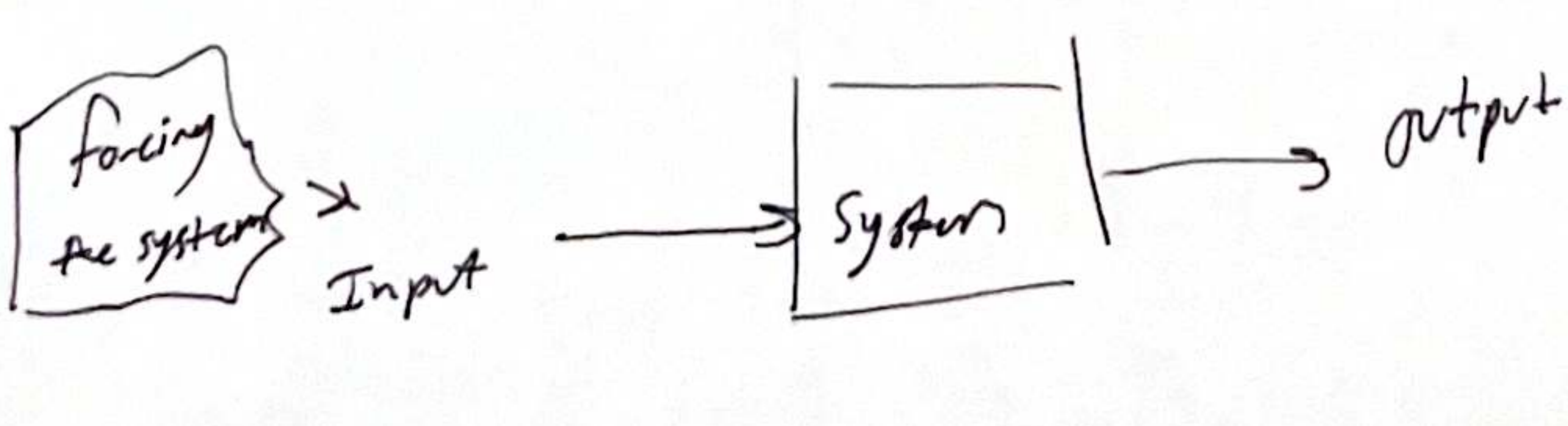
this is the weighing term for the expected value of $f(t)$

Assume f is under exponential distribution.

↳ Why would we assume such a thing?

↳ What's so important about Signal survivability & Exponential distributions?

Why weigh a Signal under exponential distribution? Why not normal distribution?



$$\boxed{e^{-x}} \quad \boxed{\mathcal{L}\{f\}(s)=\int_0^\infty f(t)e^{-st}\,dt}$$

$$\boxed{f(x) = \lambda e^{-\lambda x}}$$

1st Exponential mirrors

that rate of change proportional to amount present.

↳ It sees things through the lens of decay

Why exponentials to start with.

$\frac{d}{dx} e^{x} = e^{x}$ — 1

decay is entropy - 2

Exponentials are eigenfunctions of linear time invariant systems.

e^{-st} as a kernel diagonalises the system

↳ Allowing to solve algebraically.

The exponential provides the body. But what about the soul of that thing?

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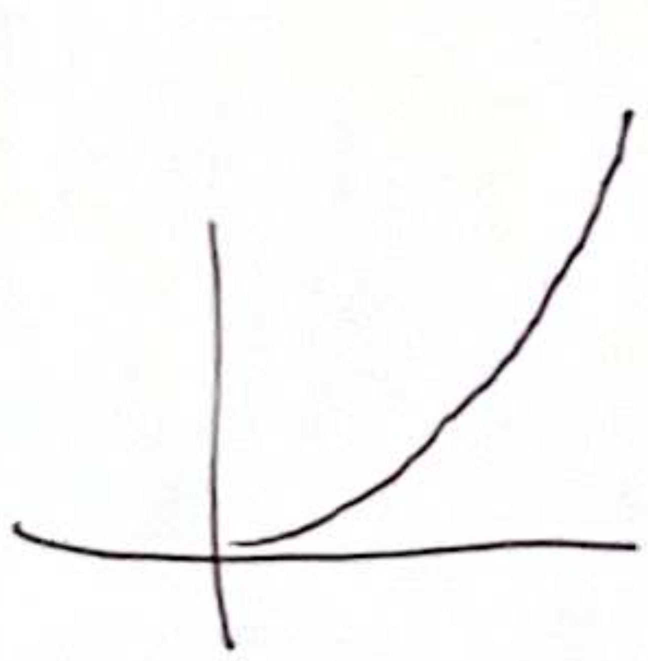
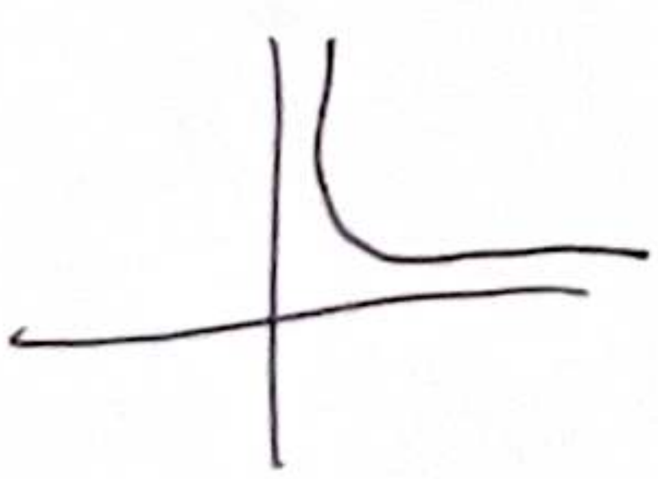




$$e^{i\theta} = \cos\theta + i\sin\theta$$

Fourier transform space

↳ stay in L^2

↳ $e^{i\omega t}$ (imaginary exponentials)

↳ $\sin(x)$, $\cos(x)$

form complete orthonormal basis in L^2 → Square integrable

~~Normed~~

~~Vector Sp~~

Vector Space } function space → Complete vector space where every element converges to a limit within that space.

↓

Normed vector space } Banach Space

$\{x_n\} \in V$

s.t.

$\forall \varepsilon > 0$

$\exists N \in \mathbb{N}$

s.t.

$\|x_n - x_m\| < \varepsilon$

$\forall n, m > N$

↓

Normed vector space with inner product } Hilbert Space

$\cos(x) \searrow \mathrm{Re}$

$\sin(x) \downarrow \mathrm{Im}$

↘ $e^{i\theta}$

↓

form the complete basis for L^2

$$\overbrace{e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)}^{\text{Orthogonal bases to express any function}}$$

→ Fourier Decomposition.

↘ Fourier Series

Laplace Transform is Very Different from Fourier Transform.

$\sin\theta$, $\cos\theta$ do so

$e^{i\theta}$ is a compact way of encoding $\sin\theta$ & $\cos\theta$ → But Laplace transform does not have only $e^{i\omega t}$

It has e^{-st} where $\boxed{s = a + i\omega}$

↘ FT projects on $\overbrace{\cos\theta, \sin\theta}^{e^{i\theta}}$ forming complete ⊥ basis.

FT $e^{i\theta}$ diagonilizes LTIs

LT use e^{-st} which have $\mathrm{Re}(e^{-st})$ & fails to form an orthogonal basis

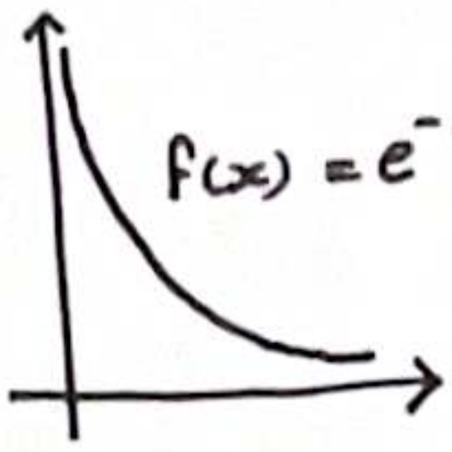
There's life, birth, and growth

Exponential Growth

$f(x) = e^{x}$

There's death, extinction, and decay

Exponential Decay



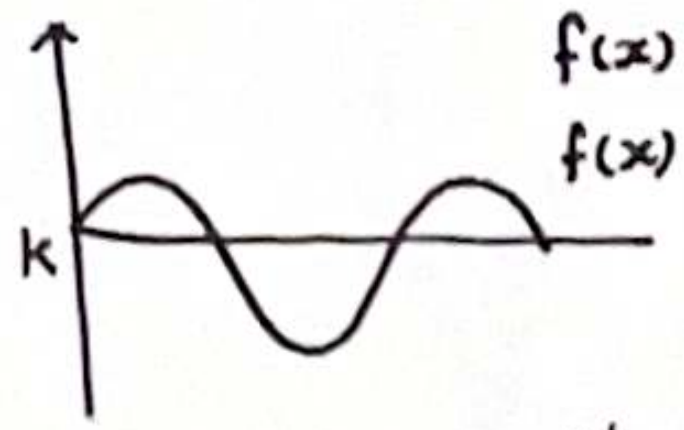
$f(x) = e^{-x}$

I am not that wise. These are the only patterns I can see.

I do not understand any non-linear behaviour

Then there's just being.

Sometimes, I feel life is just a series of constant ups and downs when nothing happens.

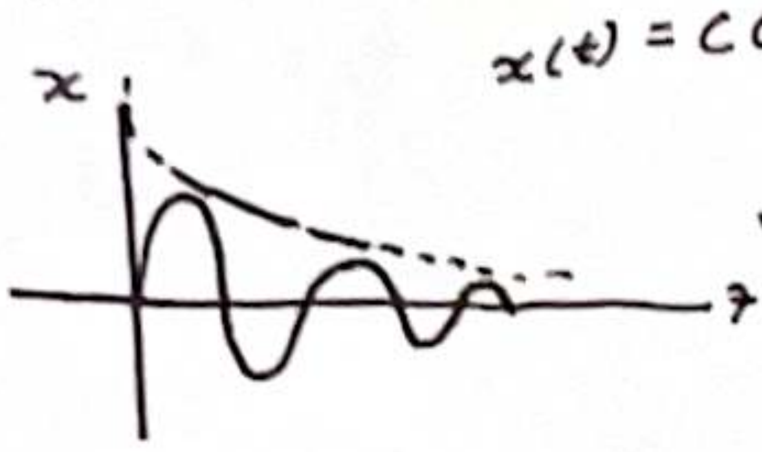


$f(x) = \sin(x)$

$f(x) = \cos(x)$

$x(t) = ce^{-\gamma t}\cos(\Omega t + \phi)$

Sometimes, we go about Spiralling up or down

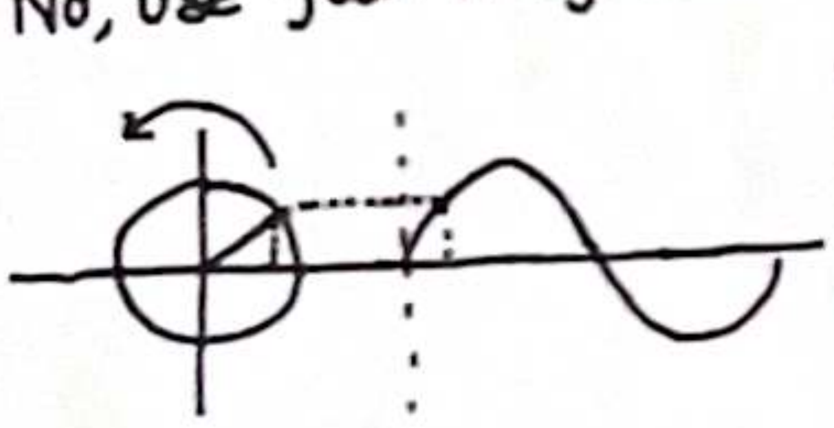


Damped Harmonic Oscillator

Well, my life is more colourful than just $e^{\pm x}$, $\sin(x)$, $\cos(x)$, and their combinations.

So are these the rules governing my life? Exponential Growth/Decay & Oscillations?

No, use your imagination, you'll reach the nucleus.



$$\boxed{e^{i\theta} = \cos(\theta) + i\sin(\theta)}$$

Oscillations on the real line Correspond to rotations on the complex plane.

That the genius and beauty of exponentials.

When restricted to $\mathbb{R}$ they capture only growth & decay.

But including complex arguments Presents us with rotations, oscillations and a dance of all between.

$$\boxed{\begin{aligned} s &= a + i\omega \\ e^{-st} &= e^{-at} \cdot e^{-i\omega t} \end{aligned}}$$

No. I refuse. My life is much more than a Series of simple math formulae. My behaviour is a more intricate function. Look at any song, tell me if it is just a Series of rising & falling sines & cosines.

You are right! I was just talking about the building blocks.

~~Sin(x) and Cos(x), imag~~

Complex exponentials form a ~~complete form a~~ Complete orthonormal basis of $L^2([-\pi,\pi])$.

Fourier.