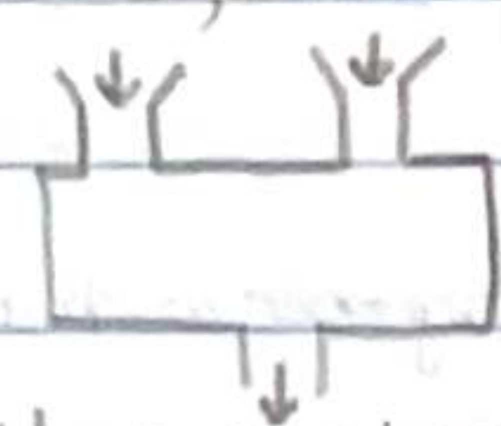
# Group (Maths)

A group is a set equipped with a binary operation which combines any two elements to form a third element in such a way that 4 conditions called group axioms are satisfied, namely closure, assossiativity, identi and invertibility.



Binary Operation - Calculation that combines two elements (operands) to produce another element. A binary operation is an operation of arity 2.

Group Isomorphism - It is a function between 2 groups that sets up a one - to - one correspondence between the elements of the grop in a way that respects the given group operations.

Isomorphic groups have the same properties and need not be distinguished.

Given 2 groups $(G, *)$ and $(H, \odot)$, A group isomorphism from $(G, *)$ to $(H, \odot)$ is a bijective group homomorphism from $G$ to H

Group $G$ is closed under binary operation $*$

It is a function $h : G \to H$ such that for all $u$ and $v$ in $G$ it holds that

$$h(u * v) = h(u) \odot h(v)$$

one - to - one correspondence.

where group operation on LHS is that of $G$ and on RHS is that of $H$.

Subgroup - Given a group G under a binary operation *, a subset H of G is called a subgroup if H also forms a group under the operation *.

Automorphism - It is an isomorphism from a mathematical object onto itself. (Symmetry). It is a way of mapping an object onto itself while preserving all structures.

A normal subgroup is a subgroup that is invariant under conjugation by members of the group of which it is part
ie. a subgroup H of a group G is normal in G iff $gH = Hg$ for all g in G.

Quotient group

Cosets - A set composed of all the products obtained by multiplying each element of a subgroup in turn by one particular element of the group containing the subgroup.

Addition of angles ← Cyclic groups

Arithmetic addition.

$S^1 = \mathbb{R}/\mathbb{Z}$

$T = \mathbb{R}/\mathbb{Z}$

If G is a group and H is a subgroup of G and g is an element of G then,
gH is the left coset of H in G with respect to g and Hg is the right coset of H in G w.r.t g.
A subgroup is normal if left coset and right coset coincide and the coset form a group called quotient group.
?? For (G,+) - Abelian group
Group operation is additive

$g + G = G + g$

Abelian group - (Commutative group) It is a group in which the result of applying the group operation to two group elements does not depend on the order in which they are written. ie. the groups obey the axiom of commutativity. Abelian groups generalise the arithmetic of addition of integers.

An Abelian group is a set A, together with an operation • that combines any two elements a and b to form another element denoted a • b.

To qualify as an Abelian group, the set and operation (A, •) must satisfy five requirements known as the Abelian group axioms.

1) Closure
$\forall a, b$ in A, the result of the operation $a \cdot b$ is also in A.

2) Associativity
$\forall a, b, c$ in A, the equation $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ holds

3) Identity Element
There exists an element e in A such that for all elements a in A, $e \cdot a = a \cdot e = a$ holds.

4) Inverse Element
For each a in A, there exists an element b in A such that $a \cdot b = b \cdot a = e$ where e is the identity element.

5) Commutativity
For all a, b in A, $a \cdot b = b \cdot a$

A group in which the group operation is non-commutative is known as a non-Abelian group.

Quotient group - It is a group obtained by aggregating similar elements of a larger group using an equivalence relation that preserves the group structure.

Circle group.