Kinematics

Kinematics is a branch of classical mechanics that describes the motion of points, bodies (object), and System of bodies (groups of objects) without considering the mass of each or the forces that caused the motion.

A kinematics problem begins by describing the geometry of the system and by declaring the initial conditions of any known values of position, Velocity and/or accleration of points within the System. Then, using arguments from geometry, the position, velocity and accleration of any unknown parts of the system can be determined.

The Study of how forces act on on masses fall within Kinetics, not Kinematics

Kinematics Is used in astrophysics to describe motion of celestial bodies and collection of such bodies. In mechanical engineering, robotics and biomechanics, kinematics is used to describe the motion of Systems composed of joined parts (multi-linked Systems) Such as an engine, a robotic arm or the human skeleton.

Geometric transformations also called rigid transformations, are used to describe the movement of components in a mechanical system, simplifying the derivation of equations of motion.

Kinematic analysis is the process of measuring the kinematic quantities used to describe motion. In engineering, for instance, kinematic analysis may be used to find the range of movement for a given mechanism, and working in reverse, using kinematic synthesis to design a mechanism for a desired range of motion. In addition, kinematics applies algebraic geometry to the study of mechanical advantage of a mechanism or mechanical system.

~~Rectiline~~ Distance and displacement.

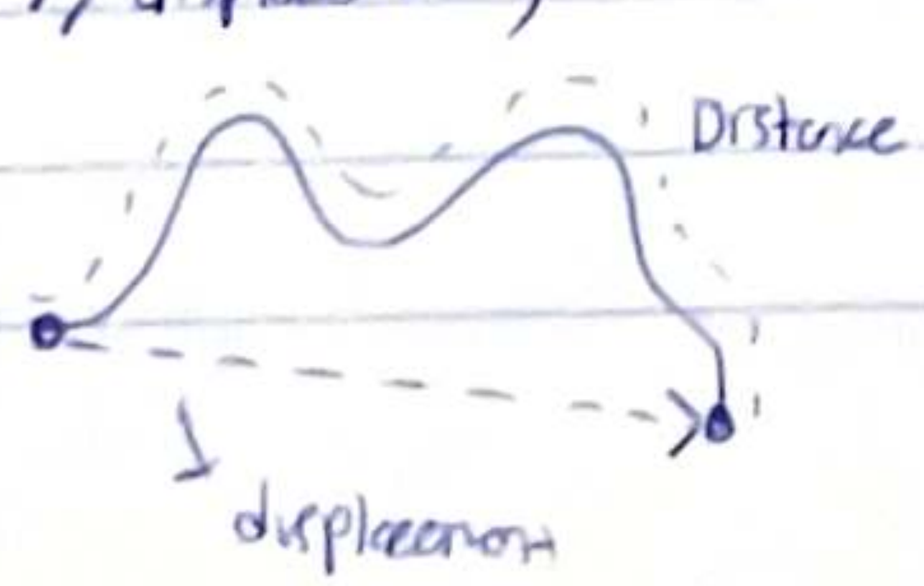
Kinematic quantities

The position vector of a particle is a vector drawn from the origin of the reference frame to the particle. (It expresses both the distance of the point from the origin and its direction from the origin.) → Displacement.

$$P = (x_P, y_P, z_P) = x_p\hat{i} + y_p\hat{j} + z_p\hat{k}$$ where x_p, y_p, z_p are the Cartesian coordinates and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the x, y and z coordinate axes, respectively. The magnitude of the position vector $|P|$ gives the distance between the point P and the origin.

$$|P| = \sqrt{x_p^2 + y_p^2 + z_p^2}$$

(Distance travelled $\geq$ displacement.)



- The direction cosine of the position vector provides a quantitative measure of direction
- The trajectory of a particle is a vector function of time, $P(t)$ which defines the curve traced by the particle

$$P(t) = x_p(t)\hat{i} + y_p(t)\hat{j} + z_p(t)\hat{k}$$

where the coordinates x_p, y_p, z_p are function of time.

Velocity and Speed.

The velocity of a particle is a vector quantity that describes the direction of motion and magnitude of motion of a particle. Mathematically, the rate of change of the position vector of a point with respect to time is the velocity of the point.

Consider the ratio of the difference of two position vectors divided by the time interval, which is called the average velocity over that time interval. This velocity is defined as $\text{Velocity} = \frac{\text{displacement}}{\text{time taken}}$.

$$\bar{V} = \frac{\Delta P}{\Delta t}$$ where ΔP is the change in position vector, Δt is the time interval.

In the limit as the time interval Δt becomes smaller and smaller, the average velocity becomes the time derivative of the position vector.

Rectilinear disp = dist.

$$\lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t} = \frac{dP}{dt} = \dot{P} = \dot{x}_p \hat{i} + \dot{y}_p \hat{j} + \dot{z}_p \hat{k}$$

Thus, Velocity is the time rate of change of position of a point. Futhermore, the velocity is tangent to the trajectory of the particle at every position the particle occupies along its path. In a non-rotating frame of reference, the derivatives of the coordinate directions are not considered as their direction and magnitude are constant.

The Speed of an object is the magnitude $|V|$ of its velocity. It is a Scalar quantity :

Angular velocity $\tilde{\omega} = \cancel{2\pi f}\ \frac{d\theta}{dt}$ angle

$$|V| = |\dot{P}| = \frac{ds}{dt}$$

Considering cyclic ~~circular~~ motion, S is the arc-length measured along the trajectory of the particle. This arc-length traveled by a particle over time is a non-decreasing quantity. Hence, $\frac{ds}{dt}$ is non-negative, which implies speed is also non-negative.

Accleration

The velocity vector can change in magnitude and in direction or both at once. Hence, the accleration is the rate of change of the magnitude of the velocity vector plus the rate of change of direction of that vector. The accleration of a particle is the vector defined by the rate of change of the velocity vector. The average accleration of a particle over a time interval is defined as the ratio :

$$\bar{A} = \frac{\Delta V}{\Delta t}$$

ΔV : difference in velocity vector

Δt : time interval

The accleration of the particle is the limit of the average accleration as the time interval approaches zero which is the time derivative

$$A = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = \dot{V} = \dot{V}_x \hat{\imath} + \dot{V}_y \hat{\jmath} + \dot{V}_z \hat{k}$$

$$\text{or } A = \ddot{P} = \ddot{x}_P \hat{\imath} + \ddot{y}_P \hat{\jmath} + \ddot{z}_P \hat{k}$$

Note: In a non-rotating frame of reference, derivative of coordinates of direction are not considered as their direction & magnitude are constants.

accleration is the 1st derivative of velocity vector and 2nd derivative of position vector of that particle.

The magnitude of acceleration of an object is the magnitude $|A|$ of its acc. vector. It is a scalar quantity:

$$|A| = |\dot{v}| = \frac{dv}{dt}$$

Relative position vectors (Relative displacement)

A relative position vector is a vector that defines the position of one point relative to another. It is the difference between the position of two points which is the difference between the components (x, y, z) of their position vector.

eg. Position of point A relative to another point B

$$P_{A/B} = P_A - P_B$$
$$= (x_A - x_B, y_A - y_B, z_A - z_B)$$

Relative Velocity

The velocity of one point relative to another is the difference between their velocities which is the difference between the component of their velocities.

$$V_{A/B} = V_A - V_B$$

This same result could be obtained by computing the time derivative of the relative position vector $\underline{\underline{R_{B/A}}}$

In the case where the velocity is close to the speed of light c (generally within 95%) another scheme of velocity called rapidity, that depends on the ratio $\frac{V}{C}$, is used in special ~~rapi~~ relativity.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \Omega^2}}$$

unitless

?!

$$\Omega = \text{rapidity} = \frac{V}{C}$$

$$1 - \Omega^2 = \frac{1}{\gamma^2} \qquad \Omega^2 = 1 - \frac{1}{\gamma^2} \qquad \Omega = \sqrt{1 - \frac{1}{\gamma^2}}$$

Relative accleration

The accleration of one point C relative to another point B is the difference between their accelerations:

$$A_{C/B} = A_C - A_B$$

The same result could be obtained by computing the second time derivative of the p.v $P_{B/C}$

Rectilinear kinematics with constant accleration.

Equation of motion

- Rigorous mathematical relationship between position, velocity and accleration.

① $V = U + at$

② $S = \frac{(U+V)}{2}t$

displacement = average velocity X time.

③ $S = Ut + \frac{1}{2}at^2$

④ $V^2 = U^2 + 2as$

Derivation.

Consider $A = \frac{dV}{dt}$

Initial conditions ~~Po and~~ position vector Po and Velocity = Vo at time = 0

$$\frac{dV}{dt} = A$$

$$V(t) = V_0 + \int_0^t A\,dt = V_0 + At$$

Second integration

$$P(t) = P_0 + \int_0^t V(t)\,dt = P_0 + \int_0^t (V_0 + At)\,dt$$

$$= P_0 + V_0 t + \frac{1}{2}At^2$$

$$P(t) - P_0 = V_0 t + \frac{1}{2}At^2$$

distance or displacement

$$A = \frac{\Delta V}{\Delta t} = \frac{V - V_0}{t}$$ Substituting in the equation before

$$P(t) = P_0 + \frac{1}{2}\left(\frac{V - V_0}{t}\right)t^2$$
$$= P_0 + \frac{1}{2}\left(V - V_0\right)t$$
$$= P_0 + \left(\frac{V - V_0}{2}\right)t \leftarrow$$

<u>Relationship between</u> <u>Velocity, Position and accleration without explicit time dependence</u> by Solving average accleration for time. (or velocity) avg. velocity

$$t = \frac{V - V_0}{A}, \quad S = \left(\frac{V + V_0}{2}\right)$$

diff

displacement

$$(P - P_0) = \text{Speed} \times \text{time}$$

$$(P - P_0) = \left(\frac{V + V_0}{2}\right) \times \frac{V - V_0}{A}$$

Δ ~~distance~~ displacement

$$(P - P_0) \cdot A = (V - V_0) \cdot \frac{V + V_0}{2}$$

diff

$$(P - P_0) = \frac{V + V_0}{2} \cdot \frac{V - V_0}{A}$$

$$(P - P_0) \cdot A = (V - V_0) \cdot \left(\frac{V + V_0}{2}\right)$$

dot product ~~or~~ is appropriate as products are scalar rather then vectors

$$= 2(P - P_0) \cdot A = (V - V_0) \cdot (V + V_0)$$

$$= 2(P - P_0) \cdot A = |V|^2 - |V_0|^2$$

α : angle between the vectors

$$= 2|(P - P_0)|\,|A| \cos\alpha = |V|^2 - |V_0|^2$$

$(P - P_0)$ → (Position P − Position 0) = Distance between 0 and P
(S)?? or displacement?? (S)

In the case of accleration always in the direction of the motion, the angle between the vectors $= 0$, So $\cos 0 = 1$ $(\alpha = 0)$

$$2|P - P_0||A| = |V|^2 - |V_0|^2$$

$$\cancel{|V|^2 = |V_0|^2 + 2|P - P_0|}$$

$$|V|^2 = |V_0|^2 + 2|A||P - P_0| \leftarrow$$

This can be simplified using the notation for the magnitude of vectors

$\|A\| = a$ $\quad \|V\| = v$ $\quad \|P - P_0\| = \Delta x$ $\quad$ where Δx can be any curvaceous path taken as the constant tangential accleration is applied across the line (displacement)

$$v^2 = v_0^2 + 2a\Delta x \leftarrow$$

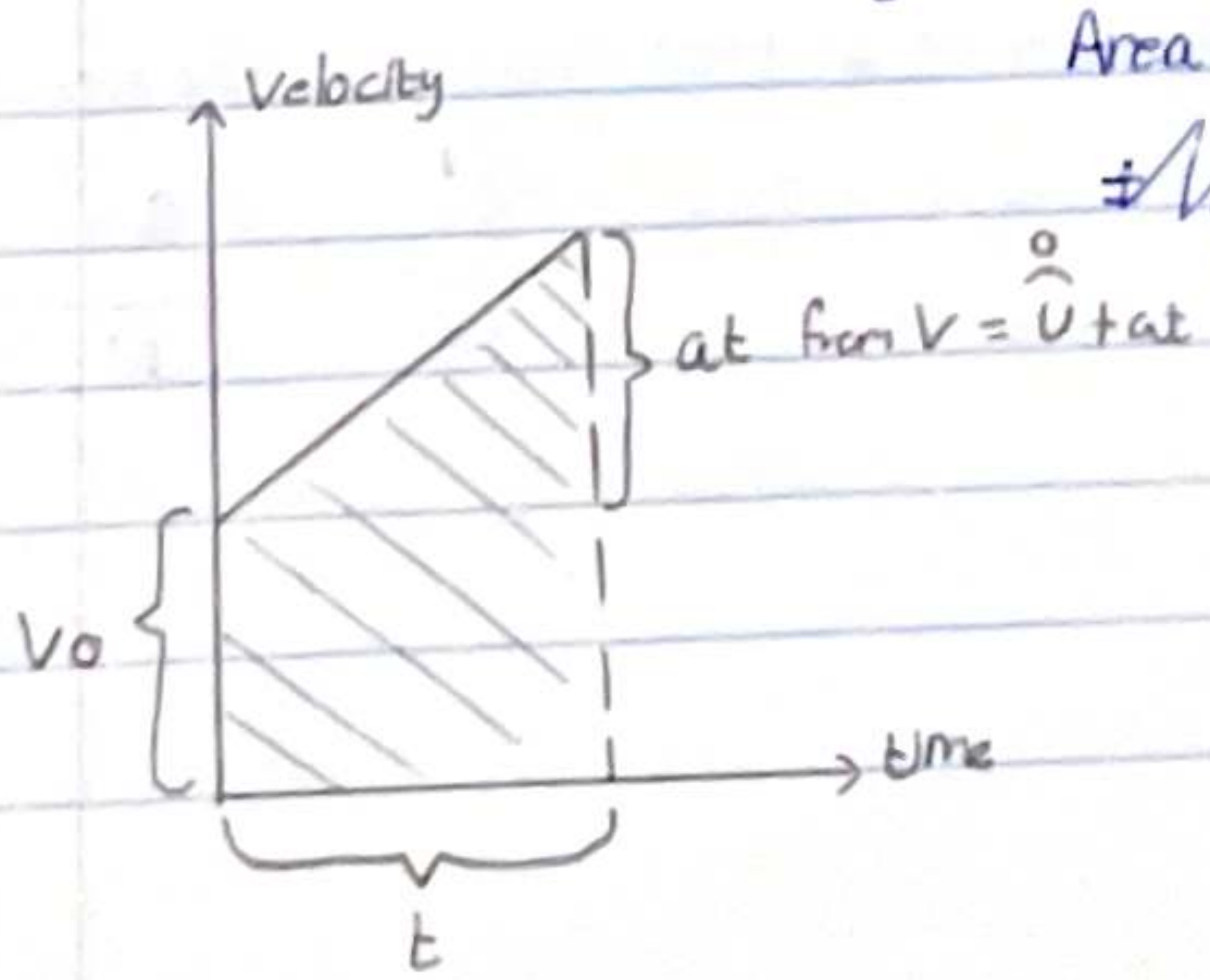
This reduces the parametric equations of motion of the particle to a Cartesian relationship of speed versus position.

This relationship is useful when time is unknown.

Derivation of $s = ut + \frac{1}{2}at^2$ via graphical method.

We know that $\Delta x = \int v \, dt$ which is the area under a $v - t$ graph.

Area under graph = Area of trapezium generated ~~(V final is not known)~~



Δx Area of trapezium $= \frac{1}{2}\left(V_0 + (V_0 + at)\right) t$

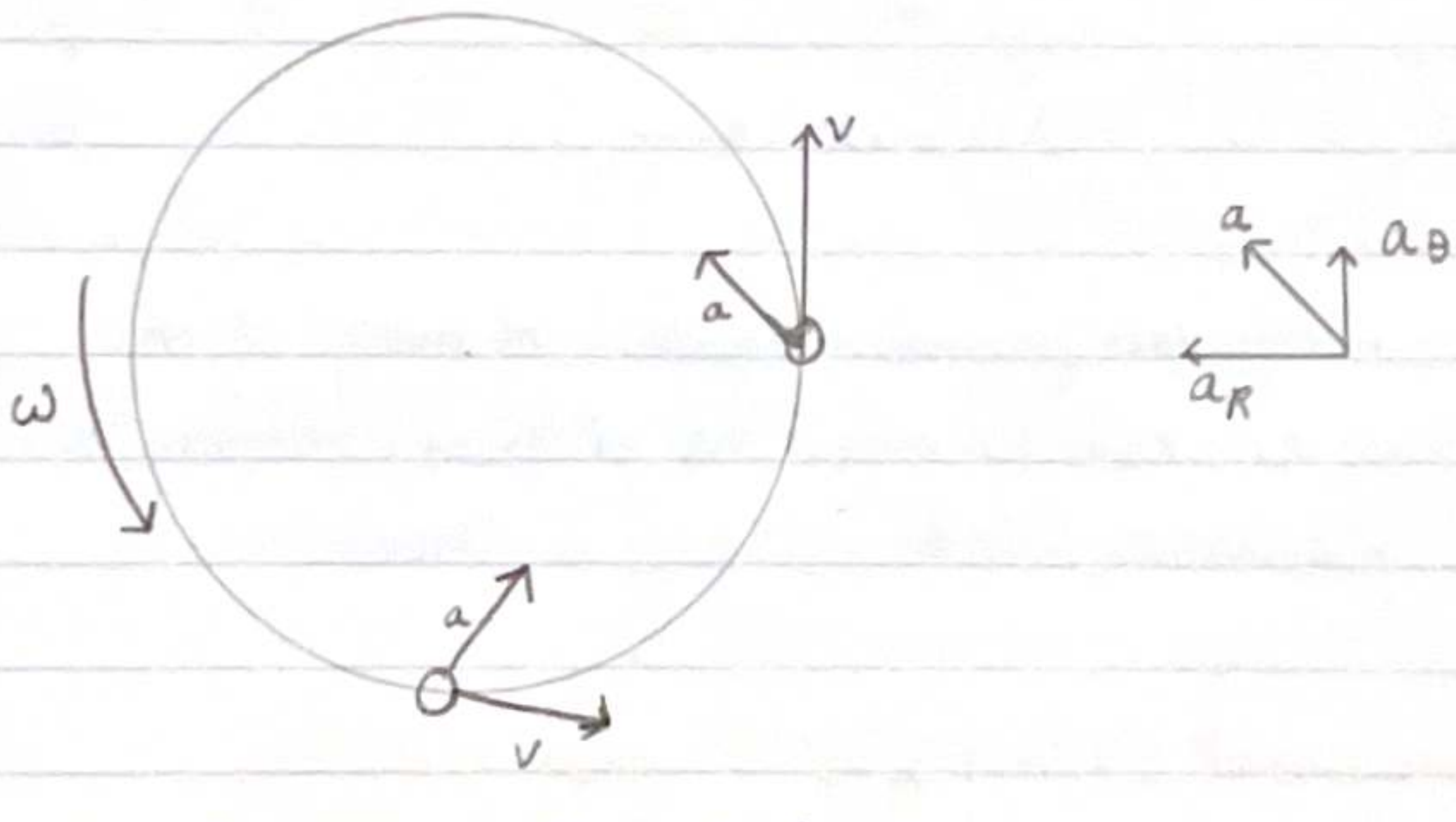
$$= \frac{1}{2}(2V_0 + at)t$$

$$\Delta x = \frac{1}{2}(2V_0 t + at^2)$$

$$= V_0 t + \frac{1}{2}at^2 \leftarrow$$

Note:
formed
displacement
distance
velocity
Speed
acceleration.

Velocity and accleration for non-uniform circular motion.



The velocity vector is tangential to the orbit, but the accleration vector is not radially inward because of its tangential component a_θ that increases the rate of rotation : ~~a~~ = $V = U + at$

$U = 0$

$V = at$

angular velocity $\frac{d\omega}{dt} = \frac{|a_\theta|}{R}$

angular acc.

$d = V \times t$

$V = \frac{d}{t}$

Kinematic Constraints. (examples)

(Kinematic Coupling.)

exactly constrains all 6 degrees of freedom.

- Rolling without Slipping

  An object that rolls without slipping obeys the condition that the velocity of its centre of mass is equal to the cross-product of its angular velocity with a vector from the point of contact to the centre of mass

  $$V_G(t) = \Omega \times r_{G/O}$$

  For the case that an object does not tip or turn, this reduces to $V = r\omega$

- Friction ??

- Inextensible cord.

  This is the case where bodies are connected by an idealised cord that remains in tension and cannot change length. The constraint is that the sum of the lengths of all segments of the cord is the total length and accordingly, the time derivative of this sum is zero. This is a dynamic but not a kinematic problem.
  equilibrium

Kinematic diagram

A kinematic diagram illustrates the connectivity of links and joints of a mechanism or machine rather than dimensions or shapes of the parts. Often links are presented as geometric objects, such as lines, triangles or squares that support schematic versions of the joints of the mechanism or machine.

Kinematic quantities

- Time : Time is a scalar quantities and it is measured in seconds. All kinematic quantities are related to time
- Distance : Distance is a scalar quantity measured in metres and is the length of the path corved.
- Displacement is the distance covered in a specific direction (vector quantity) measured in m
- Speed is defined as the rate of change of distance with respect to time and it is a scalar measured in m/s
- Velocity is the rate of change of displacement with respect to time and is a vector quantities measured in m/s.
- Accleration or deceleration can be regarded as the rate of change of velocity with respect to time. Measured im m/s^2.

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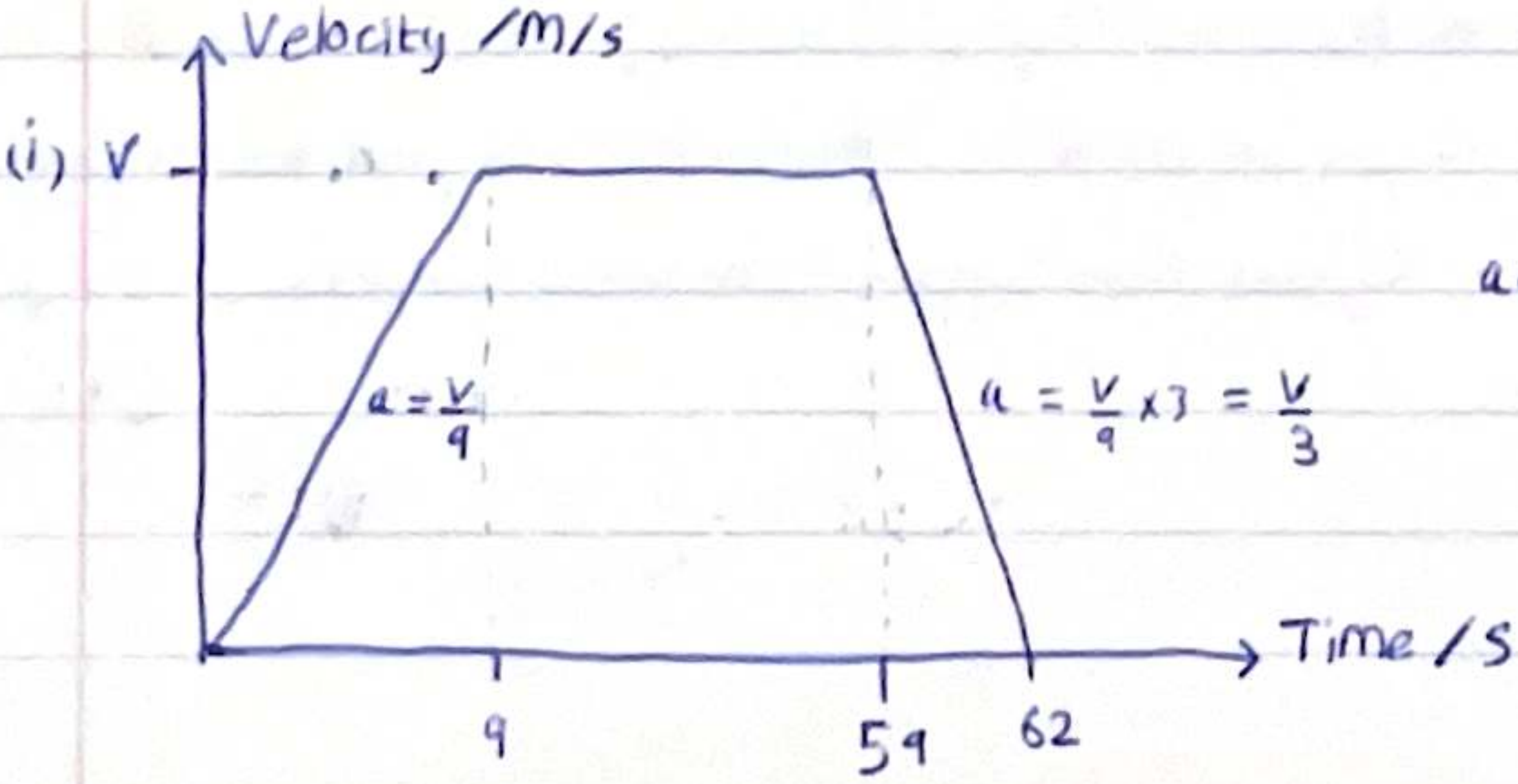
28. A motorist starting from rest accelerates uniformly to a speed of V m/s in 9s. He maintains this speed for another 50 s and then applies the brakes and decelerates uniformly to rest. His deceleration is numerically equal to 3 times his previous accleration.

(i) Sketch a velocity time graph
(ii) Calculate the time during which the deceleration takes place.
(iii) Given the total distance moved is 840 m, calculate the value of V
(iv) Calculate its initial accleration.

(i)



acc. in 9 s