Physics
English
L.T
Vectors.

$$\int e^{3x}\,dx$$
$$= \frac{1}{3}e^{3x} + C$$
(Integrate w.r.t S)

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

Laplace Transform

Time domain to frequency domain.

$$\mathcal{L}\{f(t)\} \rightarrow F(s)$$

function : mapping of set of no. on another set of no.
transform : mapping of a function on another function

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt.$$

Ex.1. $f(t) = 1$

$$\mathcal{L}\{1\} = \int_0^{\infty} e^{-st}\,dt$$

$$= \lim_{A\to\infty} \int_0^{A} e^{-st}\,dt \quad \text{(Integrate w.r.t t)}$$

$$= \lim_{A\to\infty} \left[-\frac{1}{s}e^{-st}\right]_0^A$$

$$= \lim_{A\to\infty} \left[-\frac{1}{s}e^{-sA} - -\frac{1}{s}\right]$$

Assuming $s > 0$

As $A \to \infty$, $e^{-sA} \to 0$

$$= \left[-\frac{1}{s}(0) + \frac{1}{s}\right] = \frac{1}{s}$$

$$\Rightarrow \mathcal{L}\{1\} = \frac{1}{s}$$

Ex 2. $\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st} e^{at}\, dt$

$$= \int_0^\infty e^{at-st}\, dt$$

$$= \int_0^\infty e^{(a-s)t}\, dt$$

$$= \cancel{\frac{1}{a-s}\int_0^\infty e^{(a-s)t}}$$

$$= \frac{1}{a-s}\left[e^{(a-s)t}\right]_0^\infty = \frac{1}{a-s}\Big[$$

Assume, $(a-s) > 0$, $\int_0^\infty e^{(a-s)t}$ will diverge $\Rightarrow$ No limit.

$(a-s) < 0$

$a < s$

$\cancel{\lim_{t\to\infty}}$ $\cancel{\frac{1}{a-s}}$

As $t \to \infty$, $e^{(a-s)t} \to 0$

$$\Rightarrow \frac{1}{a-s}\left[0 - e^0\right]$$

$$= \frac{1}{a-s} \times -1$$

$$= -\frac{1}{a-s} = \frac{1}{s-a}$$

$$\Rightarrow \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

LATE

Ex 3. $\mathcal{L}\{\sin at\} = \int_0^{\infty} e^{-st} \sin(at)\, dt$

$\frac{d}{dt}(uv) = u'v + uv'$

Apply integrals

$uv = \int u'v + \int uv'$

$\int u'v = uv - \int uv'$

wrt t $\left[\begin{array}{ll} u' = e^{-st} & v' = a\cos(at) \\ u = -\frac{1}{s} e^{-st} & v = \sin(at) \end{array} \right.$

Applying integration by parts,

$\int_0^{\infty} e^{-st} \sin(at)\, dt = -\frac{1}{s} \sin(at) e^{-st} - \int_0^{\infty} -\frac{a}{s} \cos(at) e^{-st}$

put $y = \int_0^{\infty} e^{-st} \sin(at)$

$y = -\frac{1}{s} \sin(at) e^{-st} - \int_0^{\infty} -\frac{a}{s} \cos(at)\, e^{-st}$

$= -\frac{e^{-st}}{s} \sin(at) + \frac{a}{s} \int_0^{\infty} \cos(at)\, e^{-st}\, dt$

Consider $\int_0^{\infty} \cos(at) e^{-st}\, dt.$

$u' = \cos(at) \quad v' = -s e^{-st}$
$u = \frac{\sin at}{a} \quad v = e^{-st}$

$u' = e^{-st} \quad v' = -a\sin(at)$
$u = -\frac{1}{s} e^{-st} \quad v = \cos(at)$

~~$\frac{1}{D} = \frac{e^{-st}}{a} \sin(at) - \int$~~

$\frac{1}{D} = -\frac{e^{-st}}{s} \cos(at) - \int_0^{\infty} \frac{1}{s} e^{-st} a \sin(at)$

$= -\frac{e^{-st}}{s} \cos(at) - \frac{a}{s} \left[\underbrace{\int_0^{\infty} e^{-st} \sin(at)}_{y} \right]$

~~$y + \frac{a}{s} y = -\frac{1}{s} \sin(at) e^{-st}$~~

$y = -\frac{e^{-st}}{s} \sin(at) + \frac{a}{s} \left(-\frac{e^{-st}}{s} \cos(at) - \frac{a}{s} y \right)$

$y = -\frac{e^{-st}}{s} \sin(at) \cancel{+} - \frac{e^{-st} a}{s^2} \cos(at) - \frac{a^2}{s^2} y$

$$y + \frac{a^2}{s^2} y = \frac{-e^{-st}}{s} \sin(at) - \frac{e^{-st}}{s^2} \cos(at) \Big|_0^{\infty}$$

$$\cancel{= \left[e^{-st} \left(-\frac{1}{s} \sin(at) - \frac{1}{s^2} \cos(at) \right) \right]_0^{\infty}}$$

$$\cancel{= \left[0 - (1(\right.}$$

$$= \left[-e^{-st} \left(\frac{a}{s} \sin(at) + \frac{a}{s^2} \cos(at) \right) \right]_0^{\infty}$$

$$\boxed{e^0 = 1 \quad -e^0 = -1}$$

As $t \to \infty$, $-e^{-st} \to 0$

$$= \left[0 \overset{+}{-} 1 \left(0 + \frac{a}{s^2} \right) \right]$$

$$\frac{y}{1} + \frac{a^2 y}{s^2} \quad \times s^2$$

$$\frac{s^2 y + a^2 y}{s^2} = \frac{s^2 + a^2}{s^2} y$$

$$= 0 + \frac{a}{s^2}$$

$$= \frac{1}{s^2}$$

$$y + \frac{a^2}{s^2} y = \frac{a}{s^2}$$

$$\left(\frac{s^2 + a^2}{s^2} \right) y = \frac{a}{\cancel{s^2}} \times \frac{\cancel{s^2}}{s^2 + a^2} = \frac{a}{s^2 + a^2}$$

$$\times \frac{s^2}{s^2 + a^2}$$

$$\Rightarrow \mathcal{L}\{\sin at\} = \frac{a}{s^2 + a^2}$$

EX4. Prove $\mathcal{L}\{\cos(at)\} = \dfrac{S}{S^2+a^2}$

u'v

$$\mathcal{L}\{\cos(at)\} = \int_0^\infty e^{-St}\cos(at)\,dt$$

Put $y = \int_0^\infty e^{-St}\cos(at)\,dt$

$U = -\frac{1}{S}e^{-St}$ $\quad V = \cos(at)$

$U' = e^{-St}$ $\quad V' = -a\sin(at)$

$$y = -\frac{1}{S}e^{-St}\cos(at) - \int_0^\infty \frac{a}{S}e^{-St}\sin(at)\,dt$$

$$= -\frac{1}{S}e^{-St}\{\cos(at) - \frac{a}{S}\int_0^\infty e^{-St}\sin(at)\,dt$$

Consider : $\int_0^\infty e^{-St}\sin(at)$

From (Ex.3)

$$\int_0^\infty e^{-St}\sin(at) = -\frac{1}{S}e^{-St}\sin(at) - \int_0^\infty -\frac{a}{S}\cos(at)e^{-St}$$

$$= -\frac{1}{S}e^{-St}\sin(at) + \frac{a}{S}\underbrace{\int_0^\infty \cos(at)e^{-St}\,dt}_{y}$$

$$y = -\frac{1}{S}e^{-St}\cos(at) - \frac{a}{S}\left(-\frac{1}{S}e^{-St}\sin(at) + \frac{a}{S}y\right)$$

$$= -\frac{1}{S}e^{-St}\cos(at) + \frac{a}{S^2}e^{-St}\sin(at) - \frac{a^2}{S^2}y$$

$$y + \frac{a^2}{S^2}y = \left[e^{-St}\left(-\frac{1}{S}\cos(at) + \frac{a}{S^2}e^{-St}\sin(at)\right)\right]_0^\infty$$

$$\frac{s^2+a^2}{s^2}\,y = \left[e^{-st}\left(-\frac{1}{s}\cos(at)+\frac{a}{s^2}\,e^{-st}\sin(at)\right)\right]_0^\infty$$

$$\lim_{A\to\infty}\left[e^{-st}\left(-\frac{1}{s}\cos(at)+\frac{a}{s^2}\,e^{-st}\sin(at)\right)\right]_0^A$$

$$= \left[0 - 1\left(-\frac{1}{s}+0\right)\right]$$

$$= \left[0+\frac{1}{s}\right]$$

$$= \frac{1}{s}$$

$$\frac{s^2+a^2}{s^2}\,y = \frac{1}{s}$$

$$\times\ \frac{s^2}{s^2+a^2}$$

$$y = \frac{1}{s}\times\frac{s^2}{s^2+a^2}$$

$$= \frac{s}{s^2+a^2} \leftarrow$$

$\mathcal{L}\{c_1 f(t) + c_2 g(t)\}$

$$= \int_0^\infty e^{-st}\left(c_1 f(t) + c_2 g(t)\right) dt$$

$$= \int_0^\infty \left(c_1 e^{-st} f(t) + c_2 e^{-st} g(t)\right) dt$$

$$= c_1 \int_0^\infty e^{-st} f(t)\, dt + c_2 \int_0^\infty e^{-st} g(t)\, dt$$

$$= c_1 \mathcal{L}\{f(t)\} + c_2 \mathcal{L}\{g(t)\} \leftarrow$$

∴ Laplace transform is a linear operator

u'v $\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)\, dt$

Applying integration by parts,

$$\int uv' = uv - \int u'v$$

$u = e^{-st}$ $\quad v = f(t)$

$u' = -se^{-st}$ $\quad v' = f'(t)$

keep bounds.

$$\int_0^\infty e^{-st} f'(t) = \left[e^{-st} f(t)\right]_0^\infty - \int -se^{-st} f'(t)$$

$\cancel{\mathcal{L} f'(t) = \mathcal{L} f(t)}$ X

$= \cancel{e^{-st} ff}$

$$\int_0^\infty e^{-st} f'(t) = \left[e^{-st} f(t)\right]_0^\infty + s\int_0^\infty e^{-st} f(t)$$

$$\int_0^\infty e^{-st} f'(t) = \left[e^{-st} f(t)\right]_0^\infty + s\int_0^\infty e^{-st} f(t)$$

Consider : $\left[e^{-st} f(t)\right]_0^\infty$

$$\lim_{A\to\infty}\left[\underbrace{e^{-st}}_{0}\ \underbrace{f(t)}_{\infty}\right]_0^A \qquad e^{-st} = 0 \leftarrow \frac{1}{e^\infty} \qquad \frac{1}{e^{st}} \to \infty \qquad \frac{1}{\infty} = 0$$

$$f(\infty) = \infty$$

convergence & divergence

Assuming $f(t)$ diverges slower that (e^{-st} converges to 0.) ?

$$\Rightarrow \left[0 - (1 \cdot f(0))\right]$$

$$= -f(0)$$

$$\Rightarrow \int_0^\infty e^{-st} f'(t) = -f(0) + s\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$$

$$\mathcal{L}\{f''(t)\} = s\mathcal{L}\{f'(t)\} - f'(0)$$

$$= s\left(s\mathcal{L}\{f(t)\} - f(0)\right) - f'(0)$$

$$= s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)$$

To find $\mathcal{L}\{f'''(t)\}$

Find $\cancel{\mathcal{L}\{\cos t\}}$

$\cancel{f(x) = \sin x}$
$\cancel{f'(x) = \cos(x)}$

$\cancel{\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)}$

Show that $\mathcal{L}\{\cos at\} = \dfrac{s}{s^2+a^2}$

$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$

$f'(t) = \cos at$
$f(t) = \dfrac{1}{a}\sin at$

$\mathcal{L}\{\sin(at)\} = \dfrac{a}{a^2+s^2}$

$\therefore \mathcal{L}\{\cos(at)\} = s\mathcal{L}\{\frac{1}{a}\sin(at)\} - \overset{0}{\frac{1}{a}\sin a(0)}$

$= \dfrac{s}{a}\mathcal{L}\{\sin(at)\}$

$= \dfrac{s}{\cancel{a}} \times \dfrac{\cancel{a}}{a^2+s^2}$

$= \dfrac{s}{a^2+s^2} \leftarrow$

$\mathcal{L}\{1\} = \frac{1}{s}$ $\quad \mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$

$s\mathcal{L}\{f(t)\} = \mathcal{L}\{f'(t)\} + f(0)$

$\mathcal{L}\{f(t)\} = \frac{1}{s}\left(\mathcal{L}\{f'(t)\} + f(0)\right\}$

Find $\mathcal{L}\{t\}$ $(f(t) = t)$ $\quad$ Diff. of $t = 1$.

$\mathcal{L}\{t\} = \frac{1}{s}\left(\mathcal{L}\{f(1)\} + 0\right)$

~~$f'(t) = 1$~~
D of $t = 1$

$= \frac{1}{s}\left(\frac{1}{s}\right)$

$= \frac{1}{s^2} \leftarrow$

Find $\mathcal{L}\{2t^2\}$

$\mathcal{L}\{2t^2\} = \frac{1}{s}\left(\mathcal{L}(2t) + (0)^2\right)$

$= \frac{1}{s}\left(2\mathcal{L}(t)\right)$

$= \frac{1}{s}\left(2 \times \frac{1}{s^2}\right)$

$= \frac{2}{s^3} \leftarrow$

$$\mathcal{L}\{t^3\} = \frac{1}{s}\left(\mathcal{L}\{3t^2\} + \overbrace{(2\times(0)^3)}^{0}\right)$$

$$\mathcal{L}(t^3) = \frac{1}{s}\left(3\mathcal{L}\{t^2\}\right)$$

$$= \frac{1}{s}\left(3 \cdot \frac{2}{s^3}\right)$$

$$= \frac{6}{s^4}$$

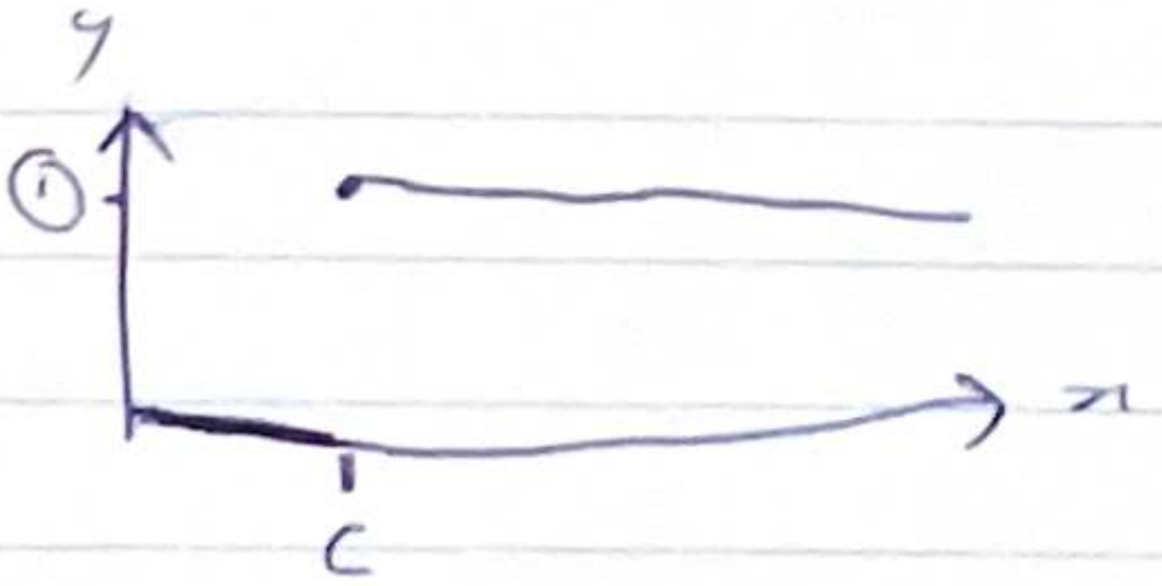
$$\therefore \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \leftarrow$$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$$

$$y'' + 5y' + 6y = 0$$

Unit Step function $\leftarrow$

$$U_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$$



Shifting of functions

$f(t)$

$2 -$

$g(x) = f(x - \text{hor. shift}) + \text{vert. shift}$

$g(x) = (f(x+2) + 5$

$y'' + 5y' + 6y = 0 \qquad y(0) = 2$

$y'(0) = 3$

$$\mathcal{L}\{y''\} = s\mathcal{L}\{y'\} - y'(0)$$
$$= s\left(s\mathcal{L}\{y\} - y(0)\right) - y'(0)$$
$$= s^2\mathcal{L}\{y\} - sy(0) - y'(0)$$

$\cancel{s\mathcal{L}\{y'\} - y'(0)}$

$$\mathcal{L}\{y''\} + 5\{y'\} + 6\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 5(s\mathcal{L}\{y\} - y(0)) + 6\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} - s\overset{2}{y(0)} - \overset{3}{y'(0)} + 5s\mathcal{L}\{y\} - 5\overset{2}{y(0)} + 6\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} - 2s - 3 + 5s\mathcal{L}\{y\} - 10 + 6\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} + 5s\mathcal{L}\{y\} + 6\{y\} - 2s - 13 = 0$$

$\mathcal{L}\{y\}(s^2+5s+6)-2s-13=0$

$\mathcal{L}\{y\}(s^2+5s+6) = 2s+13$

$\mathcal{L}\{y\} = \dfrac{2s+13}{(s^2+5s+6)}$ $\quad y = \mathcal{L}^{-1}\left\{\dfrac{2s+13}{s^2+5s+6}\right\}$

Consider : s^2+5s+6

$(s+2)(s+3)$

$\mathcal{L}\{y\} = \dfrac{2s+13}{(s+2)(s+3)}$

Split into partial fractions

$\dfrac{2s+13}{(s+2)(s+3)} = \dfrac{A}{(s+2)} + \dfrac{B}{(s+3)}$

$2s+13 = A(s+3) + B(s+2)$

put $s=-2$

$-4+13 = +A$

$A = +9$

$\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$

Put $s=-3$

$-6+13 = -B$

$7 = -B$

$B = -7$

$-a = 2$

$a = -2$

$\Rightarrow \mathcal{L}\{y\} = \dfrac{9}{s+2} + \dfrac{-7}{(s+3)}$

$= 9\left(\dfrac{1}{s+2}\right) - 7\left(\dfrac{1}{s+3}\right)$

$= 9(e^{-2t}) - 7(\cancel{4}\,e^{-3t})$

$\mathcal{L}^{-1}(9e^{-2t} - 7e^{-3t})$

$\therefore y = 9e^{-2t} - 7e^{-3t} \leftarrow$

Laplace transform of a non-homogeneous equation.

$y'' + y = \sin 2t$ Eg $y(0) = 2$

$y'(0) = 1$

$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

$\Rightarrow \mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\sin 2t\}$

$s^2\mathcal{L}\{y\} - s\overset{2}{y(0)} - \overset{1}{y'(0)} + \mathcal{L}\{y\} = \mathcal{L}\{\sin 2t\} = \frac{2}{4+s^2}$

~~$\mathcal{L}\{\sin(at)\} =$~~

put $\mathcal{L}\{y\} = Y(s)$

$\mathcal{L}\{y\}(s^2+1) - 2s - 1 = \frac{2}{4+s^2}$

~~$Y(s)\ (s$~~

$(s^2+1)Y(s) = \frac{2}{s^2+4} + 2s + 1$

$Y(s) = \frac{2}{(s^2+4)(s^2+1)} + \frac{2s}{(s^2+1)} + \frac{1}{(s^2+1)}$

Consider

$$\frac{2}{(s^2+4)(s^2+1)} = \frac{As+B}{(s^2+4)} + \frac{Cs+D}{s^2+1}$$

$$2 = (As+B)(s^2+1) + (Cs+D)(s^2+4)$$

$$2 = As^3 + As + Bs^2 + B + Cs^3 + 4Cs + Ds^2 + 4D$$

$$= (A+C)s^3 + (B+D)s^2 + (A+4C)s + B + 4D$$

$A + C = 0$ --- $A = -C$

$B + D = 0$ --- $B = -D$

$A + 4C = 0$

$B + 4D = 2$

$-C + 4C = 0$

$3C = 0$

$\therefore C = 0$

$A = 0$

$B =$

$-D + 4D = 2$

$3D = 2$

$D = 2/3$

$B = -2/3$

$$\frac{a}{s^2+a^2}$$

$$\cos(at) = \frac{s}{s^2+a^2}$$

$a^2 = s$

$s = \frac{1}{}$

$$\frac{-2}{3(s^2+4)} + \frac{2}{3(s^2+1)}$$

$Y(s)$

$$= -\frac{1}{3}\left(\frac{2}{s^2+4}\right) + \frac{2}{3}\left(\frac{1}{s^2+1}\right) + 2\left(\frac{s}{s^2+1}\right) + \frac{1}{s^2+1}$$

$Y(s) = -\frac{2}{3}$

$$= -\frac{1}{3}\sin 2t + \frac{2}{3}\sin t + 2\cos t + \sin t$$

$$\frac{5}{3}\sin t - \frac{1}{3}\sin 2t + 2\cos t$$

$$\int \frac{1}{2(y+1)} + \frac{1}{2(y-1)}\, dy = \int \frac{1}{x}\, dx$$

$$\frac{1}{2}\ln|y+1| + \frac{1}{2}\ln|y-1| = \ln|x| + C$$

$$\int \frac{1}{2(1+y)} + \frac{1}{2(1-y)}\, dy = \int \frac{1}{x}\, dx$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Let $y = e^{mx}$ be a solution

$$\frac{dy}{dx} = me^{mx} \qquad \frac{d^2y}{dx^2} = m^2e^{mx} \qquad M \text{ is a constant}$$

$$m^2e^{mx} + me^{mx} = 0$$

$$e^{mx}(m^2+m) = 0$$

$$e^{mx}\left[m(m+1)\right] = 0$$

$$M = 0 \qquad M = \underline{-1}$$

$$y = Ae^{\overset{1}{\widehat{0x}}} + Be^{-x}$$

$$= A + Be^{-x}$$

$$y = (e^{0x}, e^{-x}) = (1, e^{-x})$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0$$

$$y = e^{mx}$$

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2 e^{mx}$$

$$m^2 e^{mx} - 3(me^{mx}) - 10(e^{mx}) = 0$$

$$m^2 e^{mx} - 3me^{mx} - 10e^{mx}$$

$$e^{mx}(m^2 - 3m - 10) = 0$$

$$-10$$

$$-5 \quad +2$$

$$\downarrow$$

$$(m-5)(m+2) = 0$$

$$m = 5 \qquad m = -2$$

Auxiliary Equation

$$y = (e^{5x}, e^{-2x})$$

$$y = Ae^{5x} + Be^{-2x}$$

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

$y = e^{mx}$

$\frac{dy}{dx} = me^{mx}$

$\frac{d^2y}{dx^2} = m^2e^{mx}$

$\frac{d^3y}{dx^3} = m^3e^{mx}$

$$m^3e^{mx} - 6m^2e^{mx} + 11me^{mx} - 6e^{mx} = 0$$

$$e^{mx}(m^3 - 6m^2 + 11m - 6) = 0$$

Auxiliary Equation

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 1, 3, 2$

Soln

$m = 1$ is a root

$$\begin{array}{r|l} & m^2 - 5m + 6 \\ \hline (m-1) & m^3 - 6m^2 + 11m - 6 \\ & m^3 - m^2 \\ \hline & 0 \quad -5m^2 + 11m - 6 \\ & -5m^2 + 5m \\ \hline & 0 \quad 6m - 6 \\ & 6m - 6 \\ \hline & 0 \quad 0 \end{array}$$

$$y = (e^x, e^{2x}, e^{3x})$$

$$= Ae^x + Be^{2x} + Ce^{3x}$$

$m^2 - 5m + 6 = 0$

$-2, -3$

$m^2 - 2m - 3m + 6 = 0$

$m(m-2) - 3(m-2) = 0$ $\quad \cancel{m^2 - 5m + 6 = 0}$ $\quad -1 \times 6$

$(m-2)(m-3) = 0$ $\quad \cancel{m^2 + m - 6m + 6 = 0}$ $\quad -6 \times 1$

$m = 2, m = 3$ $\quad \cancel{m(m+1) - 6(m}$

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

y'''

$$y''' - 6y'' + 11y' - 6y = 0$$

$$\mathcal{L}[y'''] = s\mathcal{L}[y''] - y''(0)$$

$$= s(s\mathcal{L}[y'] - y'(0)) - y''(0)$$

$$= s(s(s\mathcal{L}[y] - y(0)) - y'(0)) - y''(0))$$

$$= s^3\mathcal{L}[y'']$$

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0) \quad -①$$

$$\mathcal{L}\{f''(t)\} = s\left[\underbrace{\mathcal{L}\{f'(t)\}}_{①}\right] - f'(0)$$

$$= s\left[s\mathcal{L}\{f(t)\} - f(0)\right]$$

$$= s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)$$

$$\underline{\mathcal{L}\{f'''(t)\}} = s\left[\mathcal{L}\{f''(t)\}\right] - f''(0)$$

$$= s\left[s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)\right] - f''(0)$$

$$= \underline{s^3\mathcal{L}\{f(t)\} - s^2f(0) - sf'(0) - f''(0)}$$

Laplace Transform is a linear operator.

Let $y = f(t)$

$$\mathcal{L}\{y'''\} - 6\mathcal{L}\{y''\} + 11\mathcal{L}\{y'\} - 6\mathcal{L}\{y\} = 0$$

$$s^3\mathcal{L}\{y\} - s^2y(0) - sy'(0) - y''(0) - 6\left[s^2\mathcal{L}\{y\} - sy(0) - y'(0)\right]$$

$$+ 11\left[s\mathcal{L}\{y\} - y(0)\right] - 6\left[\mathcal{L}\{y\}\right] = 0$$

$$s^3\mathcal{L}[y] - s^2y(0) - sy'(0) - y''(0) - 6s^2\mathcal{L}[y] + 6sy(0) + 6y'(0) + 11s\mathcal{L}[y] - 11y(0)$$
$$- 6\mathcal{L}[y] = 0$$

$$s^3\mathcal{L}[y] - 6s^2\mathcal{L}(y) + 11s\mathcal{L}[y] - 6\mathcal{L}[y] - y''(0) + (6-s)y'(0)$$
$$+ (-s^2+6s-11)y(0)$$

$$s^3\mathcal{L}(y) - 6s^2\mathcal{L}(y) + 11s\mathcal{L}(y) - 6\mathcal{L}(y) = y''(0) - (6-s)y'(0) - (-s^2+6s-11)y(0) = 0$$

$$\mathcal{L}(y)\left[s^3 - 6s^2 + 11s - 6\right] = y''(0) + (s-6)y'(0) + (s^2-6s+11)y(0)$$

$y''(0) =$ K

$y'(0) =$ L

$y(0) =$ M

Assumption

$$\mathcal{L}\{y\} = \frac{y''(0) + (s-6)y'(0) + (s^2-6s+11)y(0)}{s^3 - 6s^2 + 11s - 6}$$

Factorise and partial fractions

$$= \frac{y''(0) + (s-6)y'(0) + (s^2-6s+11)y(0)}{(s-1)(s-2)(s-3)}$$

Apply Inverse laplace transform.

$$\frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3} = \frac{K + (s-6)L + (s^2-6s+11)M}{(s-1)(s-2)(s-3)} = \frac{\text{Constant}}{1}$$

$A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$ X (no need for general solution)

$$= \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)}$$

$$= A\left(\frac{1}{s-1}\right) + B\left(\frac{1}{s-2}\right) + C\left(\frac{1}{s-3}\right)$$

Apply Inverse Laplace transform.

from : $\mathcal{L}[e^{at}] = \frac{1}{s-a}$ $a=1$, $a=2$, $a=3$

$$\Rightarrow Ae^{+t} + Be^{+2t} + Ce^{+3t}$$

Put $t = x$

$$\Rightarrow Ae^{+x} + Be^{+2x} + Ce^{+3x}$$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

$$y = e^{mx}$$

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

$$\frac{d^3y}{dx^3} = m^3e^{mx}$$

$$e^{i\pi} + 1 = 0$$

$$\cos\pi + i\sin\pi$$

$$= -1 + 0 + 1 = 0$$

$$m^3e^{mx} - m^2e^{mx} + me^{mx} - e^{mx} = 0$$

$$e^{mx}(m^3 - m^2 + m - 1) = 0$$

Auxilary equation.

conjugate

$\{1, i, -i\}$ Solutions $\rightarrow (m-1)(m-i)(m+i)$

Auxiliary equation has imaginary solutions.

$$y = \left(e^x, e^{ix}, e^{-ix}\right)$$

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$e^{i\pi} + 1 = 0$$

$$Ae^x + B(\cos x + i\sin x) + C(\cos(-x) + i\sin(-x))$$

$$Ae^x + B(\cos x + i\sin x) + C(\cos x - i\sin x)$$

$$Ae^x + B\cos x + Bi\sin x + C\cos x - Ci\sin x$$

$$e^{ix} = \cos x + i\sin x$$

$$Ae^x + \underbrace{(B+C)}_{D}\cos x + \underbrace{(Bi - Ci)}_{E}\sin x$$

$$e^{-ix} = \cos x - i\sin x$$

$$Ae^x + D\cos x + E\sin x$$