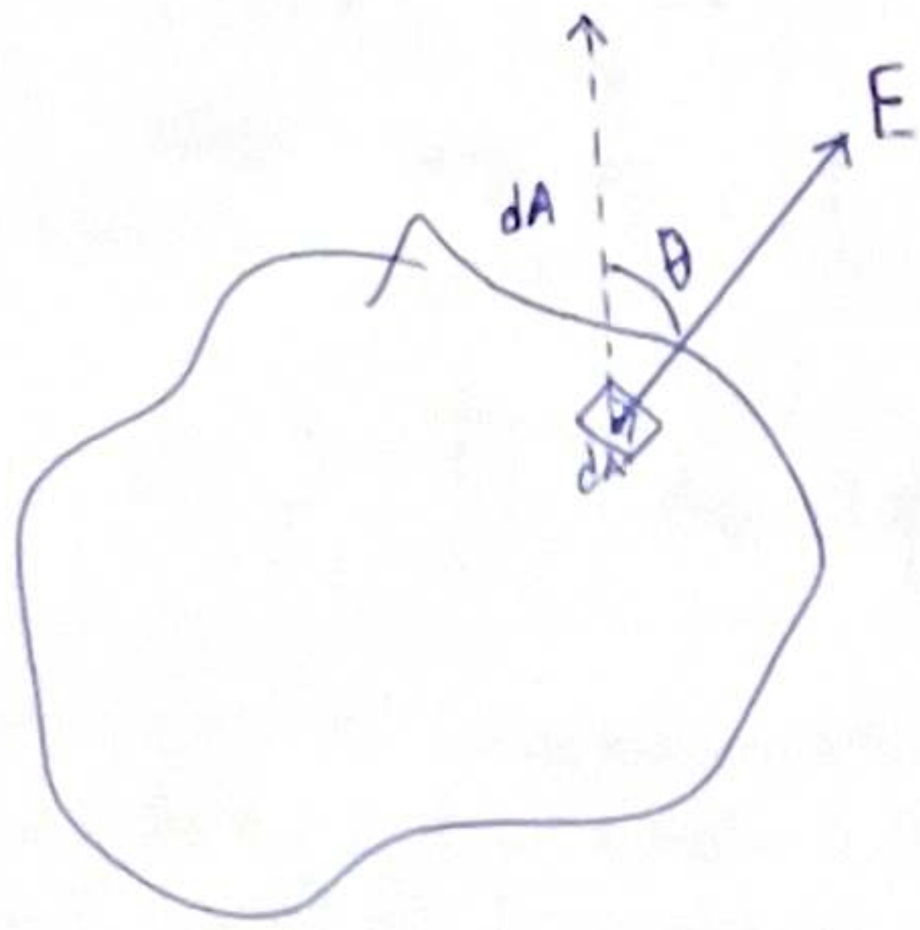
Gauss law : The net electric flux through <u>any</u> hypothetical closed Surface is equal to $\frac{1}{\varepsilon_0}$ times the net electric charge within that closed Surface.

~~Let dA be orthogonal (90°) to the~~

Closed Surface



Note : dA is a small part of the surface.

dA is a small area on the Surface

For value of component through ~~All~~ whole area,

$$\oint E \cdot dA$$

↑ This is the Surface integral.
It means that we want to sum up the value of $E \cdot dA$ for the whole surface Area.

⇒ Sum of electric flux through whole Area.

① We must find component of E orthogonal (90°) to Surface dA

Let the dotted line be 90° to dA.

Value of E projected on ~~line~~ dotted line
$= E \cos\theta$

Value of Component of E through area dA
$= |dA|\,|E| \cos\theta$

This is equivalent to a dot product

⇒ <u>Value of component of E through Area dA</u>
$= \underline{E \cdot dA}$

Note → This gives the electric flux through dA.

Let electric charge inside the object be Q, then, from Gauss' law,

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$$

Note : Net <u>electric flux</u> <u>inside</u> volume enclosed by surface is zero
① If there is no charge inside, E = 0
⑪ If +ve charge inside, E = +ve
⑪ If -ve charge inside, E = -ve.

Gauss' law states that electric charge act as source or sink for electric fields.

Consider this Sphere

with Charge ~~Q~~ Q inside (Charge enclosed in a closed Surface)

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$$

This can be treated as a multiplication Since E is Constant on all Points on the Surface → object is symmetric from Q radius is same at all points (considering Q is at the centre.

$$\underbrace{E \oint dA}_{} = \frac{Q}{\varepsilon_0}$$

Surface integral of dA gives Surface area of whole Sphere

$$A_{surface} = 4\pi r^2$$

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2} \leftarrow$$

Converting gauss' law to differential form

$\rho = \frac{Q}{V}$

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$$

Note: $Q = \rho \times$ Volume

Consider: $\frac{Q}{\varepsilon_0}$, ① rewrite right side as a volume integral (converting Q to Volume form)

$$\frac{Q}{\varepsilon_0} = \iiint \frac{\rho}{\varepsilon_0} \, dV$$

Note: ρ is the charge density (coulombs $/m^3$)

$$\Rightarrow \oint E \cdot dA = \iiint \frac{\rho}{\varepsilon_0} \, dV$$

Apply divergence theorem: It States that the flux penetrating through a closed Surface S that bounds a volume V is equal to the divergence of the field F inside the volume.

$$\Rightarrow \oint F \cdot ds = \iiint (\nabla \cdot F) \, dV$$

Apply divergence theorem to L·H·S

$$\oint E \cdot dA = \iiint (\nabla \cdot E) \, dV$$

$$\Rightarrow \iiint (\nabla \cdot E) \, dV = \iiint \frac{\rho}{\varepsilon_0} \, dV$$

Put = 0

$$\Rightarrow \iiint (\nabla \cdot E) \, dV - \iiint \frac{\rho}{\varepsilon_0} \, dV = 0$$

$$\Rightarrow \iiint \left(\nabla \cdot E - \frac{\rho}{\varepsilon_0}\right) dV = 0$$

← This integral = 0 because the only quantity for which the integral is zero is zero itself.

$$\Rightarrow \nabla \cdot E - \frac{\rho}{\varepsilon_0} = 0$$

$$\Rightarrow \nabla \cdot E = \frac{\rho}{\varepsilon_0} \leftarrow$$

Note since $D = E\varepsilon_0$

$\nabla \cdot D = \rho$

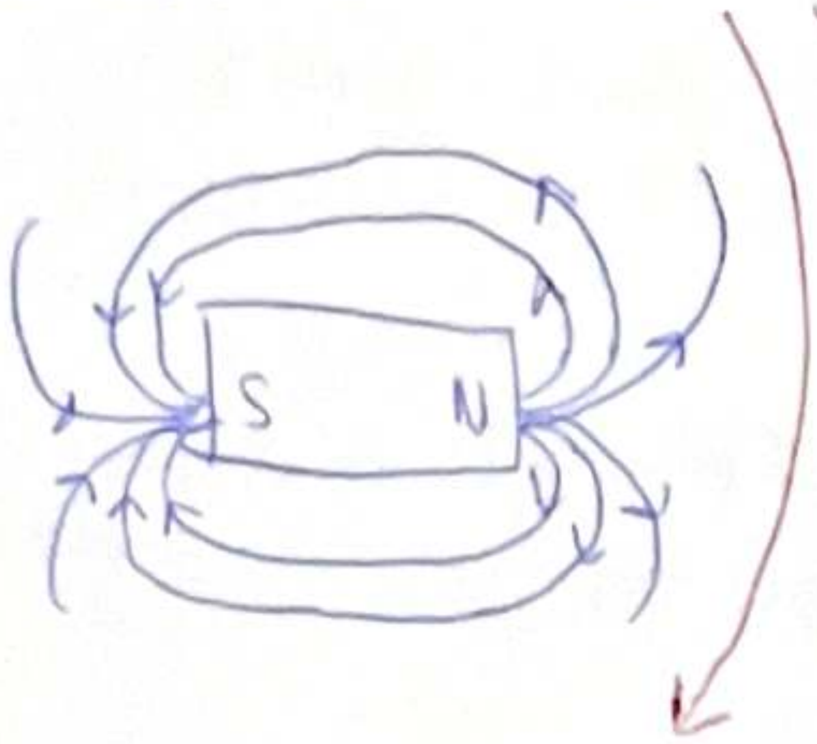
electric flux density

For point charge q which is at distance r from charge q $\Rightarrow F = Eq$

$$F \; \cancel{E} = \frac{Qq}{4\pi\varepsilon_0 r^2} \leftarrow$$

Gauss law for magnetism – The total magnetic flux passing through a closed surface is zero.

- This does not mean that no magnetic flux penetrate through the surface.
- It means that for every magnetic field line that enters the volume enclosed by the surface, there must be a magnetic field line leaving the volume. The inward (negative) magnetic flux must exactly be balanced by the outward (positive) magnetic flux. $\Rightarrow$ Net magnetic flux through a volume enclosed by a surface is zero.
- Disproves existence of magnetic monopoles.
- The net magnetic flux at a point is equal to the vector sum of all the magnetic flux at that point.



$$\oint B \cdot dA = 0$$

Converting to differential form.

Consider: $\oint B \cdot dA = 0$

Apply divergence theorem

$$\oint B \cdot dA = \iiint (\nabla \cdot B)\, dV = 0$$

Since $\iiint (\nabla \cdot B)\, dV = 0$

$\Rightarrow$ integrand $= 0$

$\Rightarrow \nabla \cdot B = 0$

Faraday's law.

Magnetic flux $\phi = B \times A$.

$\oint_s = \Sigma (B \times A)$ over whole area.

$$\phi(t) = \underset{\substack{\uparrow \\ \text{surface integral.}}}{\oint_s} B(t)\, dA$$

- The magnetic flux ϕ is the sum of B over area A.
- This eqn can be puts is a function of time.

$$EMF = \underset{\substack{\uparrow \\ \text{line integral}}}{\oint_c} d(EMF)$$

$\rightarrow$ the total EMF around the circuit is equal to summing up the small contributions at each point $d(EMF)$ around the entire length of the (closed) circuit. $\searrow$ line integral.

- Voltage is defined as the sum (integral) of electric field across a path [E is measured in V/m]

$$V = \oint_c E \cdot dL$$

[voltage between 2 points is the eqn sum of the electric field along the path between 2 points].

path of circuit

Note: Also, Differentiate with respect to 1 spacial dimension (L)

$$\frac{dV}{dL} = E \Rightarrow \underset{\substack{\uparrow \\ \text{electric field}}}{E} = \frac{dV}{dL}$$

[Electric field is a measure of how ~~much~~ fast the voltage is changing across a path].

Not used in this.

$$EMF = \oint_c d(EMF)$$

$\Vert$ same $\qquad$ same

$$V = \oint_c E \cdot dL$$

→ The differential amount of Emf at any point in the circuit is equal to the electric field at that point

$$\Rightarrow EMF_{total} = \oint_c E \cdot dL$$

Electric Field

→ Total Emf around the circuit is equal to the sum of the electric field around the whole length of the circuit.

Interpretation of Faraday's law.

- Electric current gives rise to magnetic field. Magnetic field around a circuit gives rise to a current.
- A magnetic-field changing in time $\left(\frac{d\eta}{dt}\right)$ gives rise to an electric field circulating (curl $\nabla \times E$) around it.
- A circulating electric field in time gives rise to a magnetic field changing in time.

Apply Stoke's theorem: It states that the integral of a field around a loop (line integral) is equal to integrating the curl of the field within the loop.

$$\oint_c F \cdot dL = \oint_s (\nabla \times F) dA$$

→ From R.H.S, Apply Stoke's theorem

$$\oint_c E \cdot dL = \oint_s (\nabla \times E) dA$$

$$\Rightarrow EMF = \oint_s (\nabla \times E) dA$$

$$EMF = -\frac{d\phi}{dt}, \quad EMF = \oint_s (\nabla \times E) dA, \quad \phi(t) = \oint_s B(t)\, dA$$

Lenz's law

$$\Rightarrow \oint_s (\nabla \times E) dA = -\frac{d}{dt} \oint_s B(t)\, dA$$

$$= \oint_s -\frac{dB(t)}{dt} dA$$

$$= -\oint_s \frac{dB(t)}{dt} dA$$

$$\oint_s (\nabla \times E) dA + \oint_s \frac{dB(t)}{dt} dA = 0$$

$$\oint_s \left(\nabla \times E + \frac{dB(t)}{dt}\right) dA = 0$$

Since integral $=0 \Rightarrow$ integrand $=0$

$$\longrightarrow \nabla \times E + \frac{dB(t)}{dt} = 0$$

$$\nabla \times E = -\frac{dB(t)}{dt} \quad \longleftarrow$$

Convert $\nabla \times E = -\frac{dB(t)}{dt}$ to integral form

Reverse Stoke's theorem for L.H.S

$$\oint_s (\nabla \times E) dA = \oint_c E \cdot dL$$

$$EMF = -\frac{d\phi}{dt}, \quad \phi = \oint_s B(t)\, dA$$

$$= -\frac{d}{dt} \oint_s B(t) \cdot dA$$

$$= -\oint_s \frac{dB(t)}{dt} \cdot dA$$

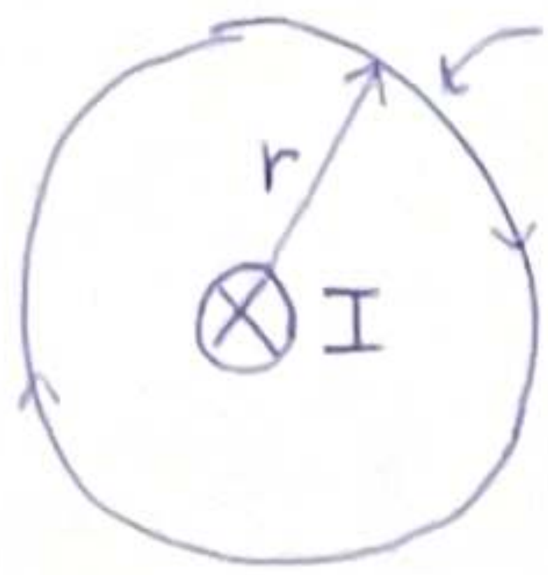
$$\Rightarrow \oint_c E \cdot dL = -\oint_s \frac{dB(t)}{dt} \cdot dA$$

Ampere's law - A conductor carrying current I produces a magnetic field which circles the wire.

$$\oint_c H \cdot dL = I_{enc}.$$

Line integral

Concider a path which encircles the wire. When the magnetic field at each point on the circular path is added, it will be numerically equal to the ~~wire~~ current in the wire.



circular path of magnetic

$\oint_c H \cdot dL$ => Sum of magnetic field along this path

H is constant along the circle at a distance r from wire.

$$\oint_c H \cdot dL = I_{enc}$$

H is a constant and H.dL can be treated as a multiplication

H : Magnetic field strength

$$\Rightarrow H \underbrace{\oint_c dL}_{} = I_{enc}$$

This gives the perimeter which is equal to the circumference of the circle of the radius r which is given by $2\pi r$

$$\Rightarrow H \times 2\pi r = I_{enc}$$

$$H = \frac{I_{enc}}{2\pi r}$$

$$\Rightarrow B = H\mu_0 = \frac{\mu_0 I_{enc}}{2\pi r}$$

$$\Rightarrow B = \frac{\mu_0 I_{enc}}{2\pi r}$$

$[\mu_0] = H/m$

$B = \frac{H}{m^2}$

$B\,m^2 = \mu_0\, m$

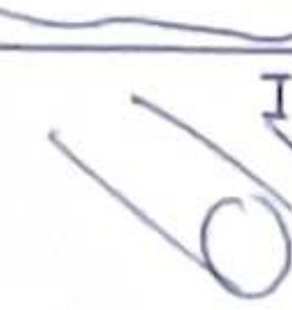
$B = \frac{\mu_0}{m}$

$H = \frac{B}{\mu_0}$

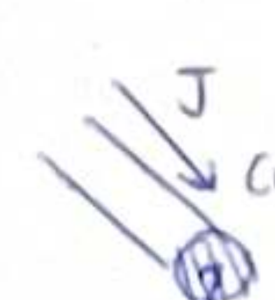
$B = H \times \mu_0$

Apply Stoke's theorem to $\oint_c H \cdot dL = I_{enc}$

Consider L.H.S $\oint_c H \cdot dL = \oint_s (\nabla \times H)\, dA = I_{enc}.$



Current I = Total flow of charge / time.



Current density J = Total flow of charge per time per cross sectional area

~~Apply Stoke's theorem to RHS~~

$$I_{enc} = \oint_s J \cdot dA$$

J is the current density => Ampere/m^2

$I = J \times A$

Convert I to Area form

Integrating J over area gives I_{enc}

Total current [I_{enc}] (I) = (Sum of current) density (J) over the (region) (dA) where current flows

$$\oint_S (\nabla \times H)\, dA = \oint_S J \cdot dA$$

* Note ; divergence of curl = 0

$$\nabla \cdot (\nabla \times F) = 0$$

$$\oint_S (\nabla \times H)\, dA - \oint_S J\, dA = 0$$

$$\oint_S [(\nabla \times H) - J]\, dA = 0$$

Since integral $= 0 \Rightarrow$ Integrand $= 0$

$$\nabla \times H - J = 0$$

$$\nabla \times H = J \leftarrow$$

If we apply $\nabla \cdot (\nabla \times F) = 0$ to this.

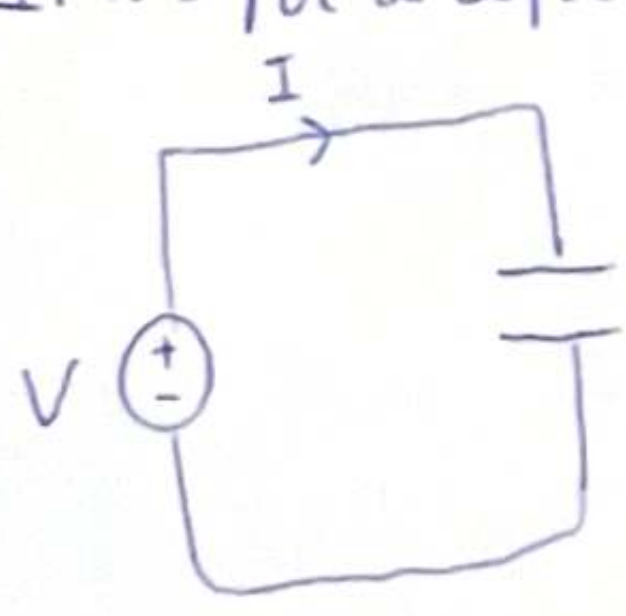
$$\nabla \cdot (\nabla \times H) = \nabla \cdot J = 0 \; (??)$$

Is the divergence of J always zero?

If the divergence of $J = 0 \Rightarrow$ the electric current flowing into a region is always equal to the electric current flowing out $\Rightarrow$ (No divergence) $\Rightarrow$ Kirchoff's first law in non reactive capacitor-Inductor circuits.

It is reasonable to think that current flow in loops (like magnetic field (w/o inductor))
but this does not work for a capacitor.

If we put a capacitor



If voltage is not constant (A.C), Current will flow through the circuit, ie. I is not equal to zero.

A capacitor is 2 conductive plate separated by air. ie. No Current should flow through air. ($\nabla \cdot J = 0$ is not valid) but it does in AC.

As current must be able to flow through air to complete circuit.

$$\nabla \cdot J = 0$$

States that the current flows through the circuit without any problem. ie. it does not diverge.

But current cannot flow through air. The current should be able to escape but it is no where to be found. It somehow is able to complete the circuit.

The divergence is not equal to zero but it cannot be calculated.

(It has been converted to Displacement current density)

Convert $\nabla \times H = J + \frac{dD}{dt}$ to integral form.

Apply Stoke's theorem to L·H·S

$$\oint_S (\nabla \times H) = \oint_C H \cdot dL$$

Consider R·H·S

$$J + \frac{dD}{dt} = \underbrace{\oint_S J \cdot dA}_{= I_{enc}} + \oint_S \frac{dD}{dt} \cdot dA$$

Not part of this

Note $= \oint_S \left(J + \frac{dD}{dt}\right) \cdot dA$?? Is this possible.

$$\Rightarrow \oint_C H \cdot dL = \underbrace{\oint_S J \cdot dA}_{= I_{enc}} + \oint_S \frac{dD}{dt} \cdot dA$$

$$\Rightarrow \underbrace{\oint_C H \cdot dL}_{\text{sum of magnetic flux in circuit}} = I_{enc} + \underbrace{\oint_S \frac{dD}{dt} \cdot dA}_{}$$

sum of displacement $\frac{dD}{dt}$, J_D or displacement current density in Area of capacitor.

Current in circuit

(Note This one does not include the one in capacitor).

Interpretation of Ampere's law.

A flowing current (J) gives rise to a magnetic field that circles the current.

A time-changing Electric flux density (D) gives rise to a magnetic field that circles electric flux density field (D field).

Implication of faraday's law and Ampere's law.

$$\nabla \times E = -\frac{DB}{dt}$$

① Faraday's law - A changing magnetic field gives rise to a circulating electric field.

② Ampere's law = A changing electric field gives rise to a circling magnetic field.

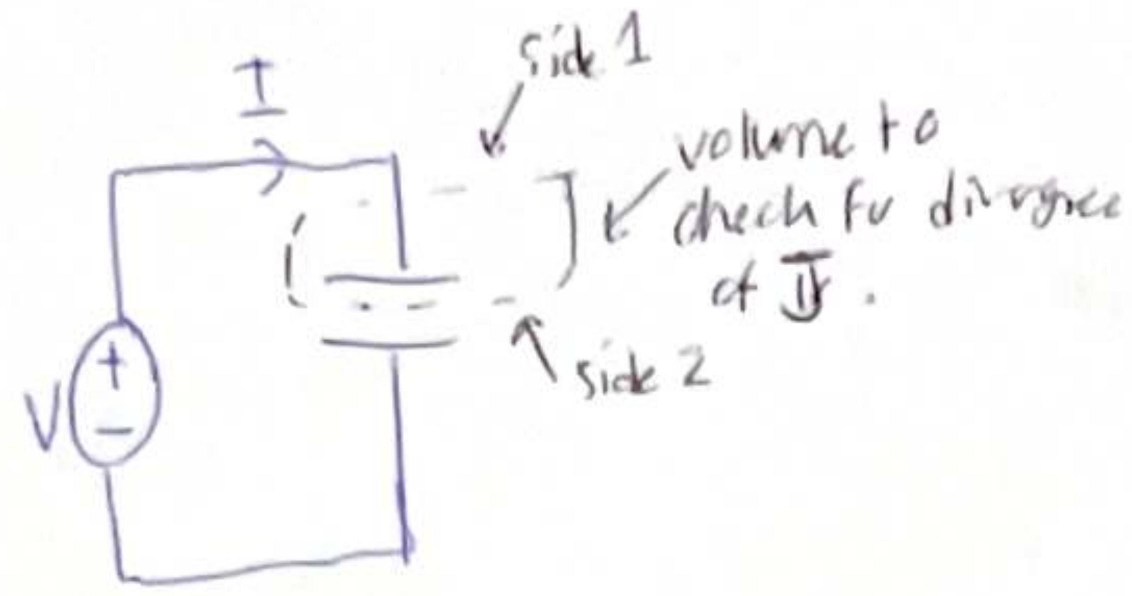
$$\nabla \times H = -\frac{dD}{dt} + J$$

- A changing magnetic field creates a ~~cir~~ ~~electric~~ circulating electric field around the changing magnetic field. The magnetic field oscillates ~~increases and decreases~~ which implies that electric field warps back and forth $\Rightarrow$ electric field is also changing with time
- A changing electric field creats a circulating magnetic field. The electric field oscillates increases and decreases which causes the magnetic field to warp back and forth around the changing electric field $\Rightarrow$ magnetic field is also changing with time.

A changing electric field creates a changing magnetic field which creates a changing electric field which creates a changing ~~magnetic~~ magnetic field which ....

$\Rightarrow$ Propagation of EM waves.

Check for divergence at capacitor.



* For $\nabla \cdot f = 0$

$\Rightarrow$ Field that leave must also enter.

$D = E\varepsilon_0$

Electric flux density

Electric field.

Current enters via Side 1 and Side 2 splits capacitor in half.
It is known that current flows through the circuit.
Current enters through Side 1 of the volume but nothing leaves it. $\Rightarrow$ *Divergence of J is not zero

$\Rightarrow \nabla \cdot J = 0$ is not valid

But electric flux density (D) is changing within the capacitor. $\rightarrow$ A·C $\rightarrow$ Current is changing with time
$\Rightarrow$ Electric field / Electric flux density also changes with time.

And, $\nabla \times E = -\dfrac{dB(t)}{dt}$ $\Rightarrow$ time-varying magnetic field gives rise to an electric field.

Could a time-varying electric field give rise to a magnetic field?

$\dfrac{dD}{dt}$ $\Rightarrow$ time varying electric field.

$$\frac{dD}{dt} = J_d \leftarrow \text{Displacement current density (Current in capacitor)}$$

Note $J_d = \dfrac{dD}{dt}$

although D is changing, it is still electric flux ie. current.

curl $\nabla \times H$ is the magnetic field rotating around the electric flux density.

$$\Rightarrow \nabla \times H = J + J_d$$

$$\Rightarrow \nabla \times H = J + \underbrace{\frac{dD}{dt}}_{=0 \text{ in}} \leftarrow$$

$=0$ in normal (No capacitor - Inductor) circuits.

$\dfrac{dD}{dt}$ is equivalent to current which is equivalent to magnetic field given by $\nabla \times H$.

$$\mathcal{E}mf = \oint_C E\, dL$$

$$\Phi(t) = \oint_S B(t)\, dA$$

lenz law

$$\mathcal{E}mf = -\frac{d\Phi}{dt} = -\frac{d}{dt}\Phi$$

$$\Rightarrow \oint_C E\, dL = -\frac{d}{dt}\oint_S B(t)\, dA$$

Apply stoke theorem

$$\oint_C E\, dL = \oint_S (\nabla \times E)\, dA$$

$$\oint (\nabla \times E)\, dA = -\frac{d}{dt}\oint_S B(t)\, dA$$

$$= -\oint_S \frac{dB(t)}{dt}\, dA$$

$$\oint (\nabla \times E)\, dA + \oint_S \frac{dB(t)}{dt}\, dA = 0$$

$$\oint \left((\nabla \times E) + \frac{dB(t)}{dt} \right) dA = 0$$

If integral $= 0 \Rightarrow$ Integrand $= 0$

$$\nabla \times E + \frac{dB(t)}{dt} = 0$$

$$\Rightarrow \nabla \times E = \frac{-dB(t)}{dt}$$

$$\nabla \times H = J + \frac{dD}{dt}$$

must also be in surface integral form

$$= \oint_S (\nabla \times H)$$

$$\oint_C H\, dL$$

$$\oint_C H\, dL = \underbrace{\oint_S J \cdot dA}_{I_{enc}} + \oint_S \frac{dD}{dt}\, dA$$

$$\oint_C H \cdot dl = I_{enc} + \oint_S \frac{dD}{dt}\, dA$$

Ampere's law → Current carrying a wire produces a circulating magnetic field

$$\oint_c H \, dL = I_{enc},$$

H is constant

$$H \oint dL = I_{enc} \implies H \times 2\pi r = I_{enc}$$

~~Radius~~ circumference (perimeter)

$$H = \frac{I_{enc}}{2\pi r}$$

$$B = H\mu_0 = \frac{\mu_0 I_{enc}}{2\pi r}$$

Apply Stoke's theorem to L.H.S

$$\oint_c H \cdot dL = I_{enc}$$

$$\oint_c H \cdot dL = \oint_s (\nabla \times H) \, dA$$

Conv. I_{enc} to Area form

$$I_{enc} = \oint_s J \, dA$$

$$\oint_s (\nabla \times H) \, dA = \oint J \, dA$$

$$\oint (\nabla \times H - J) \, dA = 0$$

$$\implies \nabla \times H = J$$

$$\nabla \cdot (\nabla \times F) = 0 \longrightarrow \text{Divergence of curl} = 0$$

$$\implies \nabla \cdot J = 0 \text{ ??}$$ Does not work in Capacitor

A.C flows in a capacitor

⇒ Electric field strength / flux is changing.

Could a time varying electric field give rise to a magnetic field?

$$\implies \frac{dD}{dt} = J_d$$ ← displacement current density.

electric flux density.

$$\implies \nabla \times H = J + \frac{dD}{dt}$$

Consider $\oint_S E \cdot dA = \frac{Q}{\varepsilon_0}$

Charge density

$Q = \rho \times V$

volume

Apply divergence theorem

$\frac{Q}{\varepsilon_0}$ $\cancel{(\nabla \cdot E)V} = \iiint \frac{\rho}{\varepsilon_0} dV$

rewrite as volume integral

$\oint_S E \cdot dA = \iiint (\nabla \cdot E) \, dV$

$\Rightarrow \iiint (\nabla \cdot E) \, dV = \iiint \frac{\rho}{\varepsilon_0} dV$

$\iiint \left(\nabla \cdot E - \frac{\rho}{\varepsilon_0}\right) dV = 0$

If integral $= 0 \Rightarrow$ Integrand $= 0$

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$D = E \times \varepsilon_0$

$\nabla \cdot D = \rho$

$\oint_S B \cdot dA = 0$

Apply divergence theorem.

$\oint_S B \cdot dA = \iiint (\nabla \cdot B) \, dV = 0$

If integral $= 0 \Rightarrow$ Integrand $= 0$

$\Rightarrow \nabla \cdot B = 0$

By extension since $B = H\mu_0$ $\Rightarrow \nabla \cdot H = 0$

$\phi = B \times A$

$\Rightarrow \phi(t) = \oint_S B(t) \, dA$ — (2)

In Series, total Emf or Voltage = Sum of all Voltage across every component.

$\Rightarrow \text{Emf} = \oint_C d(\text{Emf})$

minute emfs at all points.

Apply Stoke theorem (when converting to differential)

$\oint_C E \, dL = \oint_S (\nabla \times E) \, dA$ — (1)

Emf.

Voltage or Potential = Electric field $\times r$

$\frac{Q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 r^2} \times \underset{\text{distance}}{r} = \frac{Q}{4\pi\varepsilon_0 r}$

$\Rightarrow$ Emf / Voltage = Sum of electric field At all points

$\Rightarrow \text{Emf} = \oint_C E \, dL$

Note :

$$\oint_S F \cdot dS = \iiint (\nabla \cdot F)\, dV$$ Divergence theorem

Surface to Volume .

$$\oint_C F \cdot dL = \oint_S (\nabla \times F)\, dA$$ Stoke theorem

line to Surface .

$B = H \times \mu_0$

$D = E \times \varepsilon_0$

$J = \frac{I}{A}$ current density

Charge density . $\rho = \frac{Q}{V}$

B : Magnetic flux density &

H : Magnetic field strength (flux)

D : Electric flux density

E : electric field strength .

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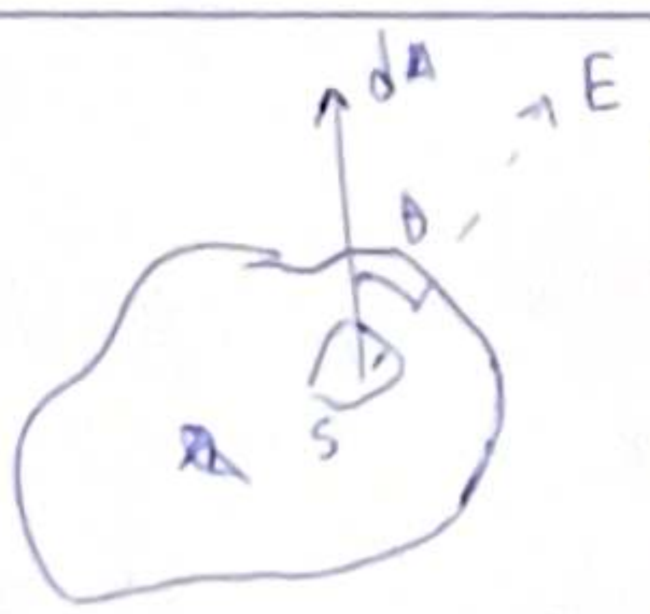
(1)



$E \cdot dA = |E||dA| \cos\theta$

component of E orthogonal to Surface S

Integrate over Surface S .

(Sphere) radius = r

E is constant on surface .

$$\oint_S E \cdot dA = \frac{Q}{\varepsilon_0}$$ charge inside (from gauss law)

$$E \underbrace{\oint_S dA}_{\text{S-A sphere} = 4\pi r^2} = \frac{Q}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$