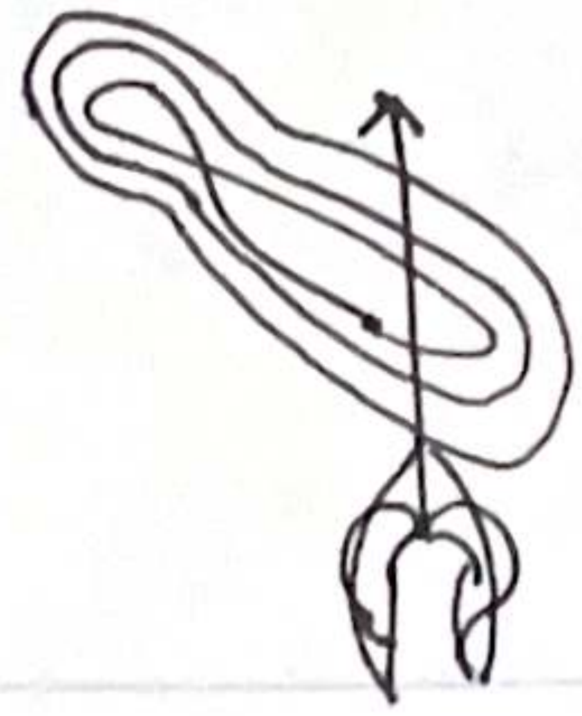
  

Limits of Sets

Let $\{A_k\}$ be a sequence of subsets of some space X. Then :

Set of Points that persists infinitely across all subsets

lim inf → ~~Outer limit set~~

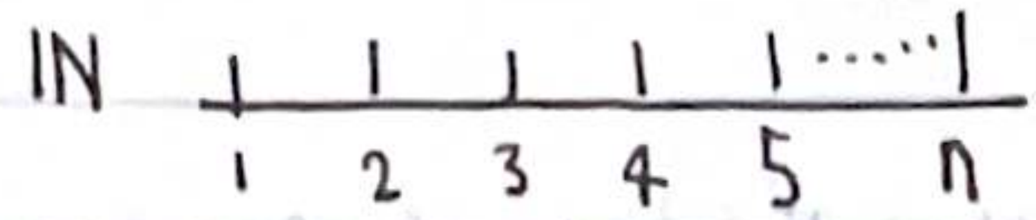
$$\boxed{\liminf_{k\to\infty} A_k = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k}$$

lim sup → lim Sup is the outer limit set

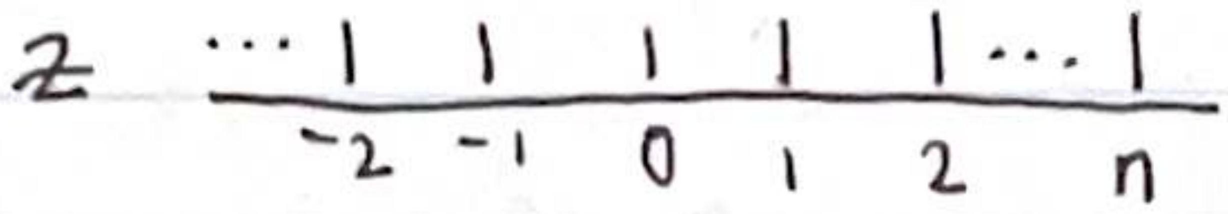
These points eventually appear

$$\boxed{\limsup_{k\to\infty} A_k \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k}$$

$\lim_{k\to\infty} A_k$ Exists if and only if $\liminf A_k$ and $\limsup A_k$ Agree



infinite of them
(Countable)



$\cancel{\exists f : \mathbb{Z} \to \mathbb{N}}$

$\exists f : \mathbb{N} \to \mathbb{Z}$ where f is one-one and onto (bijective)

$\cancel{f(n) = \{}$

Proof

Aim: Show the bijection exists

Pushforward Measure

A pushforward measure is obtained by transferring a measure from one measurable space to another using a measurable function.

Given measurable spaces (X_1, Σ_1) and (X_2, Σ_2), a measurable mapping $f: X_1 \to X_2$ and a measure

$$\mu : \Sigma_1 \to [0, +\infty],$$

the pushforward of μ is defined to be the measure $f_*(\mu) : \Sigma_2 \to [0, +\infty]$ given by

$$\boxed{f_*(\mu)(B) = \mu(f^{-1}(B)) \text{ for } B \in \Sigma_2}$$

The pushforward measure is also denoted $f_{\#}\mu$

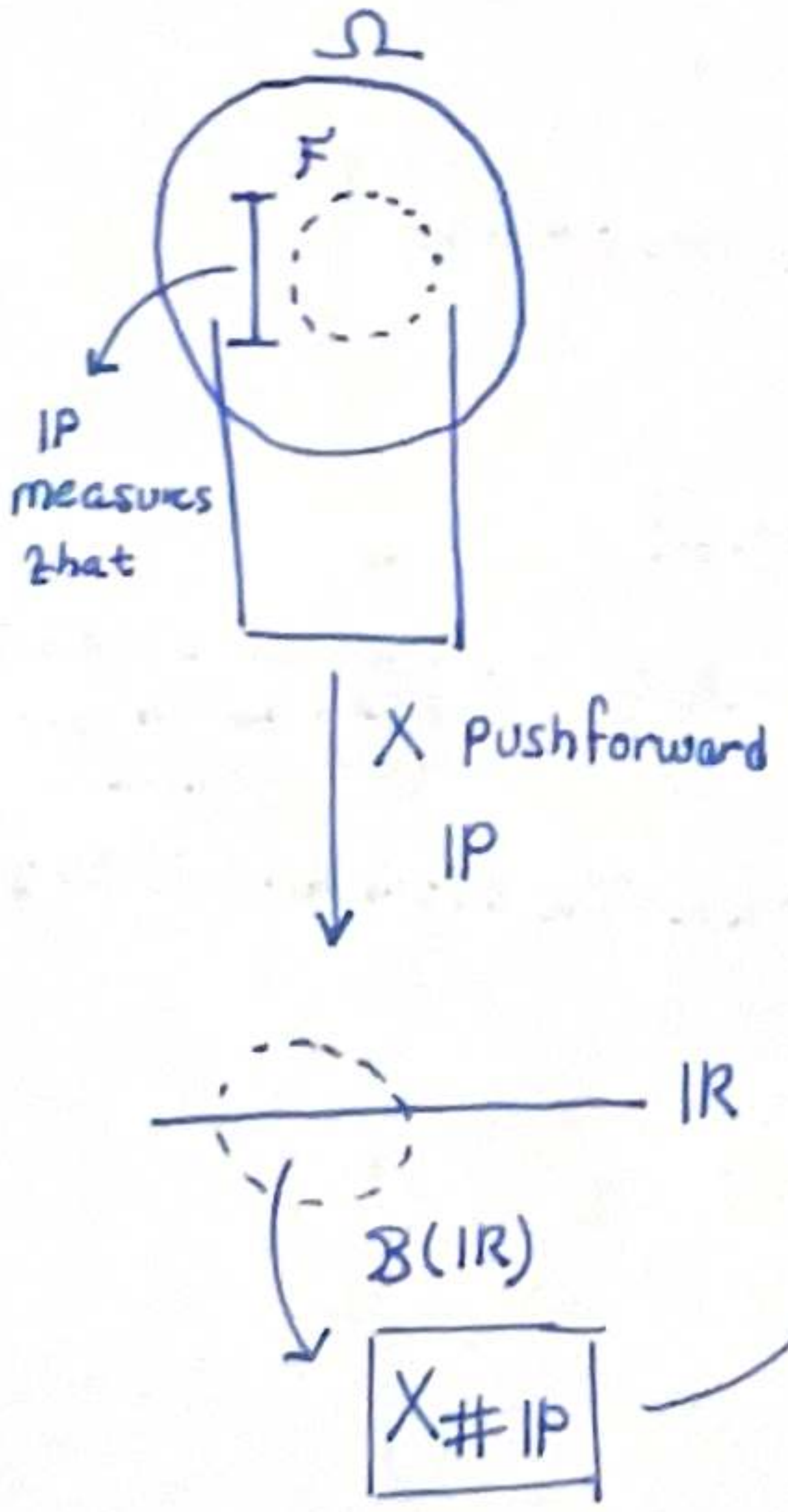
$\mathcal{F}$ ensures measure of events occur properly

this is defined as $\mathbb{P}$ on $\mathcal{F}$

In terms of probability, A random variable pushforward the probability measure $\mathbb{P}$ defined on a probability space to the real line (equipped with a Borel σ-algebra).

To also allow for measure to be defined consistently.

$$\boxed{\underbrace{\mathbb{P}_X}_{X_{\#}\mathbb{P}}(B) = \mathbb{P}(X^{-1}(B)) \text{ for } B \in \mathcal{B}(\mathbb{R})}$$



$\mathbb{P}_X$ reflects the distribution of the probability on $\mathbb{R}$.

This was evaluated by $\mathbb{P}$ involving $\mathcal{F}$ & Ω

$\mathbb{P} : \mathcal{F} \to [0, 1]$

Now, the measure $\mathbb{P}$ must also in some way involve Ω for the notion of probability to be relevant.

You need a good number system to be able to write probability density functions. The pushforward transfers that probability to that number system so it can now be captured in equations and interpreted numerically.

This is the thing that is captured by Probability density functions.

Random Variable

Measure Theory Definition

↳ A random variable is a measurable function from a probability measure space (sample space) to a measurable space.

$X : \Omega \to \mathbb{R}$ $\quad (\Omega, \mathcal{F}, \mathbb{P}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$

A measurable function is a function between the underlying sets* of two measurable spaces that preserves the structure of the space.

↘ The pre image of any measurable set is measurable.

Defn : Let (X, Σ) and (Y, T) be measurable spaces

[could be $(X, \mathcal{F})$ and $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$] for our probability cases

X & Y both have σ-algebras Σ & T respectively (defn of measurable space)

$f : X \to Y$ is said measurable if :

For every ~~E ∈ T~~ $E \in T$, the preimage of E under f is in Σ

↳ $\boxed{\forall_{E \in T} \quad f^{-1}(E) = \{x \in X \mid f(x) \in E\} \in \Sigma}$

Question : Is the domain of X, Ω or $\mathcal{F}$

↖ This is a set of sets, not outcomes

↘ The underlying set*

How does the probability measure $\mathbb{P}$ relate to the σ-algebra $\mathcal{F}$?

How does pushing forward onto $\mathbb{R}$ transfer the measure to $\mathcal{B}(\mathbb{R})$ → This allows measure to be defined as it is a σ-algebra.

The $\boxed{\text{push forward measure}}$ is called the distribution of the random variable. The distribution is thus a probability measure on the set of all possible values of the random variable.

Just because information is defined as something doesn't fit it.

∴ Not True. Not verified..

Intuition says yes but I am looking for something else..

Information Content

$$I(x) \overset{\text{def}}{=} -\log_b(Pr(x))$$

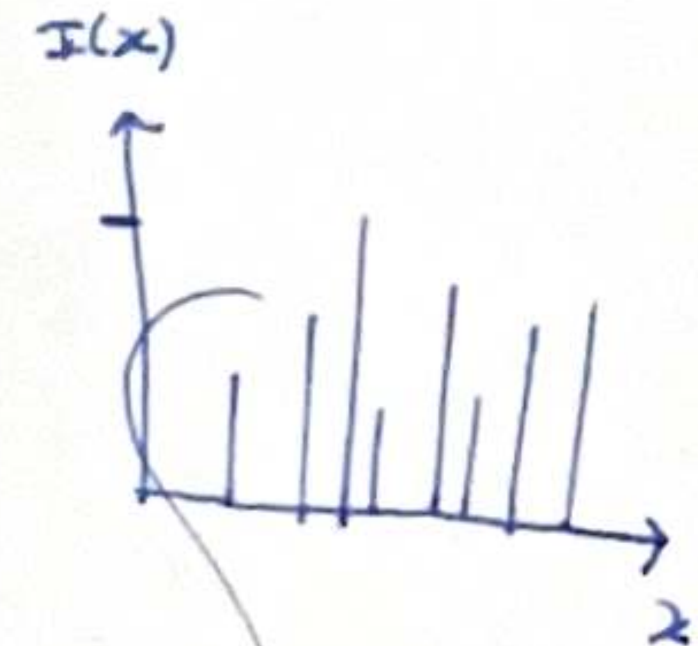
Definition of Surprisal / Information content

If an event happens certainly, $Pr = 1$

$\Rightarrow I(x) = 0$

↳ Yields no information.

If an event is rare, it is surprising that it happens.



Entropy captures the average trend of the Surprisal

↳ ~~How much inform~~

Higher Entropy means our prediction fails more, more unexpected things, more chaos

↳ Not necessarily evil, but more unknown variation

↳ Away from ideal

Given an event x in a probability Space, the event has an associated information content it brings to the space.

↳ Through a measurable function and a pushforward measure, we can interpret the probability space as a probability distribution.

The Probability Space / The probability distribution has events and events have associated information content.

↳ Entropy (Shannon) is the expected Surprisal over the distribution.

↳ [How surprising / how much information we can get from that space]

$$\boxed{S = -\sum P_i \log_b(P_i)}$$

Cosmos & chaos move in waves & the stationary point is information

To keep the definition of information fixed

Being surprised by a new discovery,

⇒ Chaos responds to the order we know by definition.

Changes according

But before discovery, that was unexpected

Still Predictable

↳ Analytic in ℂ

Predictable