Manifolds

- Generalises the concept of Surfaces

$\downarrow$

Calculus on Surfaces

e.g.

S^2 (Surface of a sphere)

Motivation: Use Calculus (Regular on open domain) on Surfaces.

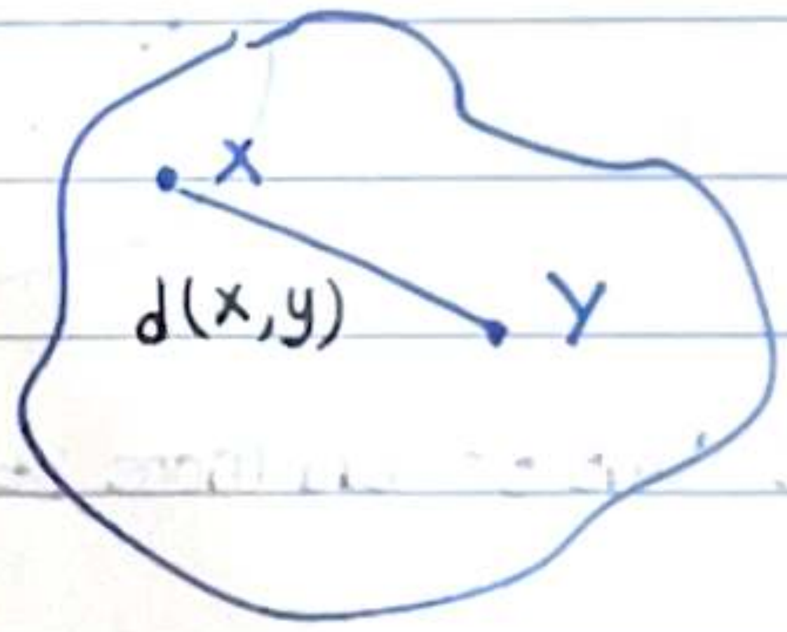
① Topology

A lot of the notions in topology are already formulated in metric spaces.

Metric Space : (X, d)

Set — distance function

e.g. Euclidean Metric (in $\mathbb{R}^2$) is $d = \sqrt{x^2+y^2}$



Start with Functional Analysis

Functional Analysis

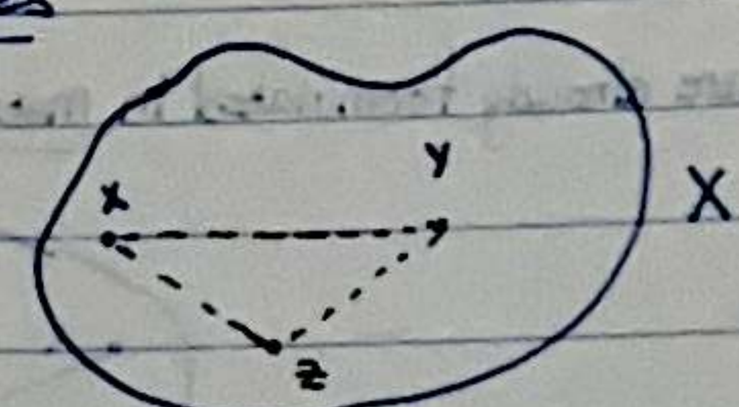
Linear Algebra

$\mathbb{R}$ & $\mathbb{C}$ Analysis

It is Analysis with linear algebra with infinite dimension.
Study of topological algebraic Structure.

Metric Spaces

Set X



We want to add more structure : distance

Metric

$d : X \times X \rightarrow [0, \infty)$

1) $d(x,y) = 0 \iff x = y$
2) $d(x,y) = d(y,x)$
3) Triangle Inequality
   ↳ If we make a detour from x to y through z, the distance gets longer.

$\boxed{d(x,y) \leq d(x,z) + d(z,y)}$

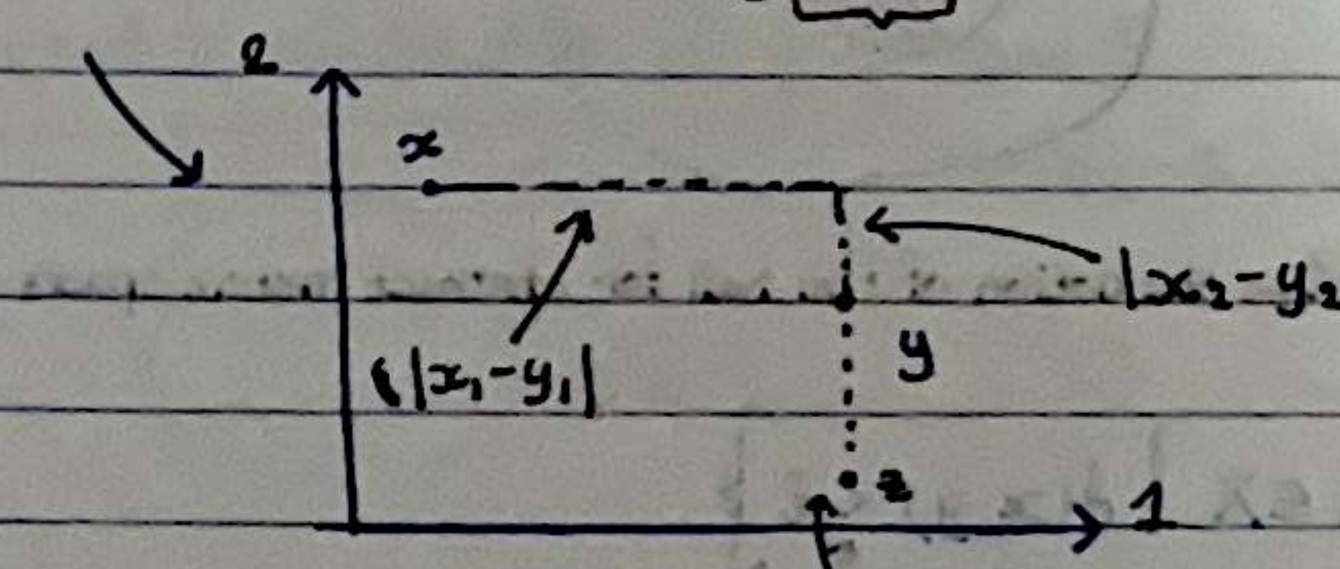
(X, d) is called a metric Space.

e.g. (a) $X = \mathbb{C}$, $d(x,y) = |x-y|$

x, & y, refer to 2 points in the first dimension.

(b) $X = \mathbb{R}^n$, $d(x,y) = \sqrt{(x_1-y_1)^2 + \dots + (x_n-y_n)^2}$ Standard Euclidean Metric

(c) $X = \mathbb{R}^n$, $d(x,y) = \max\{|x_1-y_1|, \dots, |x_n-y_n|\}$



Using this metric, z has same distance from x as y.

$\max|x_1-z_1|$ ↳ $d(x,y) = d(x,z)$

(d) X any set ($\neq \emptyset$), $d = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$ (Discrete Metric Space)

d is a metric?

1) $d(x,y) = 0$ when $x = y$ ✓
2) $d(x,y) = d(y,x)$ ✓ Symmetric defn. (Already described in the metric)
3) Δ - Inequality. ✓

$x, y, z \in X$, $d(x,y) \leq d(x,z) + d(z,y)$

If $x \neq y$ then z must be a distinct point from at least one of x or y.

First Case $X = y \Rightarrow 0 \leq \underbrace{d(x,z) + d(z,y)}_{\geq 0}$

other case $x = y = z \Rightarrow d = 0$ which is not possible as $x \neq y$.

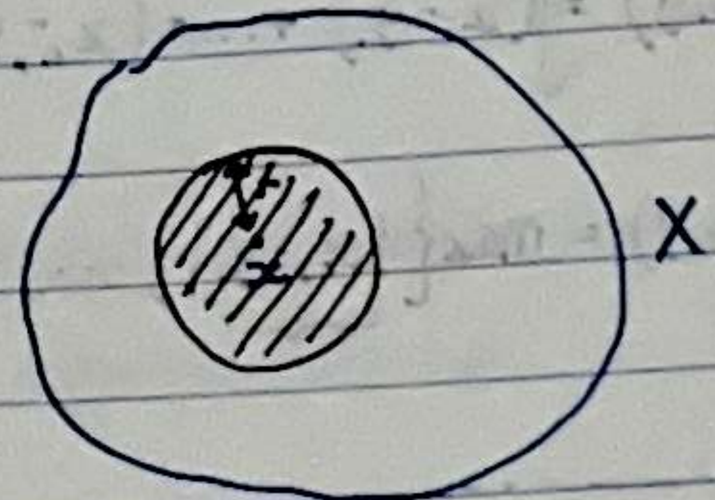
Second Case $x \neq y \Rightarrow 1 \leq d(x,z) + d(z,y)$

$= 1 \begin{cases} d(x,z) \\ \text{or} \\ d(z,y) \end{cases}$

Open and Closed Sets

- Suppose we fix point x, using our metric we find all the points at the same distance r from x.

In $\mathbb{R}^2$, this is a circle around x. In $\mathbb{R}^3$ it is a sphere or ball.



We want to generalise the notion of the ball for abstract metric spaces; $B_\varepsilon(x)$

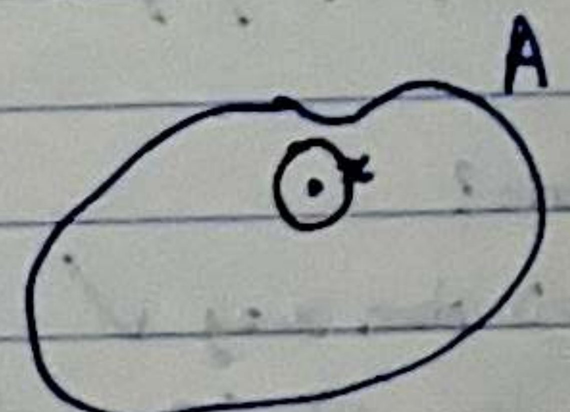
$$B_\varepsilon(x) = \{ y \in X \mid d(x,y) < \varepsilon \}$$

(Open ball of radius $\varepsilon > 0$ centred at x) Not the boundary but everything INSIDE

$B_\varepsilon(x)$ is NEVER EMPTY as atleast x lies within.

Notions:

1) Open Set: Suppose $A \subseteq X$



↳ If we are inside A, we should never see the boundary of the set A.

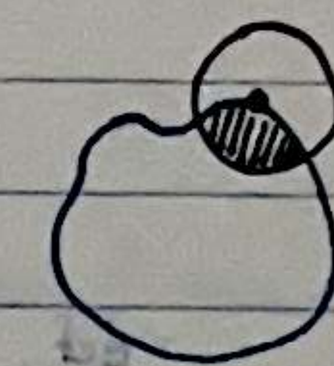
↳ We do so by choosing ε as SMALL as needed.

Do that for each point in A.

$A \subseteq X$ is called open if for each $x \in A$ there is an open ball with $B_\varepsilon(x) \subseteq A$

The closer we get to the boundary, smaller ε is needed.

2) Boundary Points



↳ This ε-ball has points both in and out of Subset A.

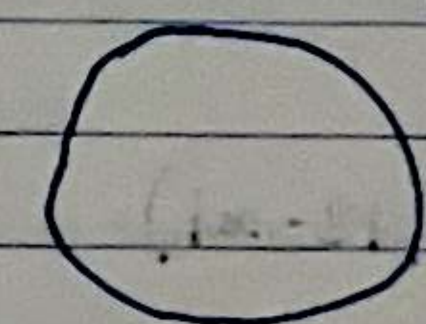
If this happens for any ε however small it is, again

$A \subseteq X$. $x \in X$ is called a boundary point for A if all open balls around x contain points in both A and A^c.

↳ $\forall \varepsilon > 0 : B_\varepsilon(x) \cap A \neq \emptyset$ and $B_\varepsilon(x) \cap A^c \neq 0$

$[A^c = X \setminus A]$

NOTE: Boundary point for Subset A can lie both in and out of A. ??

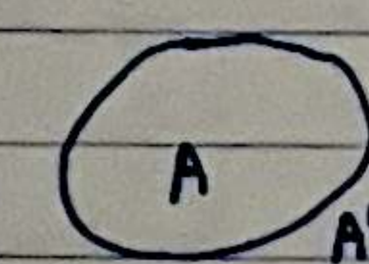


$\partial A = \{ x \in X \mid x \text{ is a boundary point for } A \}$

- An open set is a set where the boundary points are outside of A.

A is open $\Leftrightarrow A \cap \partial A = \emptyset$ [Boundaries are Excluded]

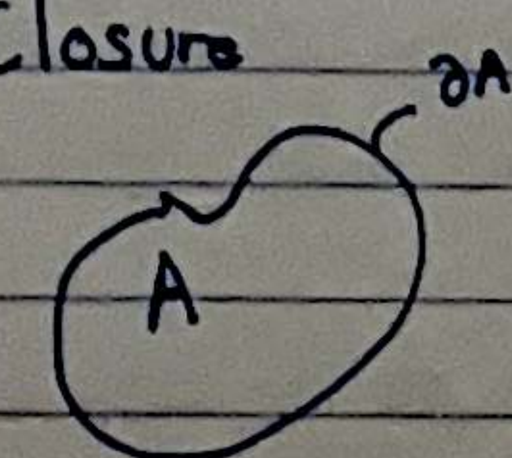
3) A is closed $\Leftrightarrow A \cup \partial A = A$



A is a closed set if $A^c = X \setminus A$ is open.

$X \setminus A$ excludes the boundary points.

4) Closure



If we start with ~~a closed~~ an arbitrary subset A we want a closed subset.

↳ Do so by: $\bar{A} = A \cup \partial A$

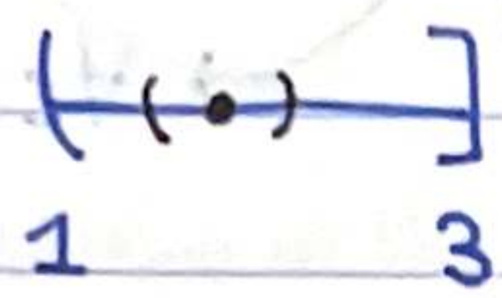
↳ Smallest closed set that still contains A.

Example:

$X \overset{\text{def}}{=} (1,3] \cup (4,\infty)$, $d(x,y) \overset{\text{def}}{=} |x-y|$, (X,d) is a metric space.

(a) $A \overset{\text{def}}{=} (1,3] \subseteq X$

Is subset A open?



For all $x \in A$, $x \neq 3$, define $\varepsilon : \min(|1-x|, |3-x|)$

Statistical Mechanics - Stanford. (I)

Motivation: Classical Physics is about predictability.
↳ We know the initial conditions & the laws of Evolution of a System.
↳ $V = U + at$ & initial velocity.
↳ We can make perfect predictions

e.g. We have a closed system
↳ External influences X
e.g. a jar of gas.
If we know the initial conditions of all the particles, with the Newtonian laws, we can predict behavior at any point in time.

↳ This predictability is useless. There are too many particles & too many to keep track of...
↳ Use the power of probability Theory & Law of large numbers
↳ Useful when we do not know the initial conditions with great precision, when ideal predictability is impossible.

Probability becomes very precise. But still not applicable to everything...

Statistical mechanics allow us to predict, $N, V, T, U, \ldots$
But sucks at predicting the position of every molecule.

e.g. fluctuation.
↳ Don't violate probability
Anyway. But very unlikely.
↳ In sealed room, high density of particles at a corner...

Probability Calculus

Ω : the Space of possibilities
e.g. Outcomes of ~~an System~~ an experiment / States of a System

Ω = Set of all i
$\{H, T\}$

Let $i \in \Omega$
↳ Possible out come
↳ e.g. H or T when flipping coin

e.g. A coin can either be H or T

Summary: $\underline{i}$ → A state of a System (H or T)

When there is ignorance about a state, assign probability
$i = 1 \ldots N$
$P(i) \geq 0$ (No −ve probability)
$\sum_i P(i) = 1$ {Sum of all possibilities} $= 1$

$\underline{P(i)}$ → The prob. that the System is in State i

Law of Large number

$\underline{N_i}$ → № of times measured System in State i

Suppose we ran our experiment N times and N_i is the no of times we obtained the i^{th} outcome,

$\underline{F(i)}$ → function assigning value to a State i

$$\boxed{\lim_{N\to\infty} \frac{N_i}{N} = P(i)}$$

The ratio converges to $P(i)$ for very large N

Let $F(i)$ be a measurable function of a State i → Energy / Momentum.
$F(H) = +1$
$F(T) = -1$ } flipping Coin

Average of $F(i)$ is a useful ~~...~~ Measure. → Average energy of a system.
$\langle F(i) \rangle$ = Average value of a function averaged over the probability distribution.
$= \sum_i F(i) P(i) \xrightarrow{\text{Sum}} F(i)$ weighted over probability

That is following the $P(i)$ obtained from $\frac{N_i}{N}$, N trials of ~~exp~~ measuring the States of the System,

We find the average $\underline{F(i)}$ (the measurable function of interest)

↳ $F(i)$ could be a function of State
e.g. V_i is the velocity of one particle of mass m in a box and F returns the energy as $F(i) = \frac{1}{2} m V_i^2$

$\langle F(i) \rangle$ Would then be the average velocity of that one particle across all systems.

~~$\langle F(i) \rangle = \sum_i F(i)$~~ $\langle F(i) \rangle = \sum_i f(i) P(i)$
↳ $f(i)$ weighted under the probability distribution.

This allows us to estimate the average value $\langle F(i) \rangle$ (which we assumed to be an average quantity that we could not measure directly)

↳ The average $F(i)$ is not a thing that we could measure directly.
e.g. Suppose we could not measure Temperature directly.
That method would allow us to estimate the average kinetic energy of particles in a system.
↳ Being temperature.

{ Macrostate → Temperature
{ Microstate → Velocity of particles

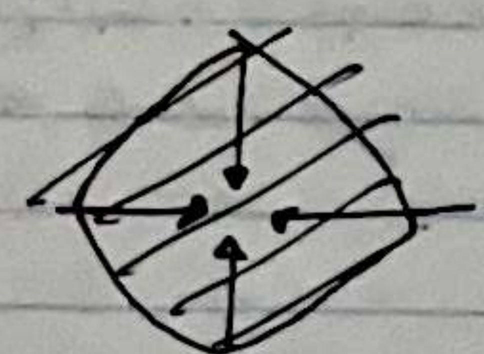
If we consider again $F(H) = +1$ $\quad P(H) = 1/2$

$F(T) = -1$ $\quad P(T) = 1/2$

$$\boxed{\langle F(i) \rangle = \sum_i F(i)\, P(i)} \quad \left(= \frac{\sum_i F(i)\, N(i)}{N} \right)$$

$\Rightarrow \langle F(i) \rangle = 0$

Not a possible state of the system. } Despite that, it is still the average Value.



$P(H) = 1/2$, $P(T) = 1/2$ } → Implies Symmetry in the System concerning the (geometric) part of it which we are exploiting when making our measurement.

A bias in the Symmetry (Weight of the coin) of the system would result in deviations from the ideal probabilities.

↳ Or present a deeper Symmetry accounting for uneven mass distribution.

↳ Finding a new System to explain that.

↳ e.g. Momentum can be spread equally (1/2 and 1/2) and that would allow us to redefine $F(i)$ to present the Skewed results 1/3 & 2/3 for example for the values H & T.

At its core, Symmetry still persists.

Dice

↳ 6 possible States.

$\{R, B, Y, G, O, P\}$

$P(R) = 1/6$ → All faces are Symmetric to each other

$P(i) = 1/6$

↳ If no symmetry, then skewed probabilities.

However, until we find a deeper symmetry, we experiment. $\frac{N_i}{N} \xrightarrow[N \to \infty]{} P(i)$

↳ We find $P(i)$

Other Method is tracing the evolution of a system to get information about a system.

↳ We observe the System and identify some rules

↳ The laws of motion of the System.

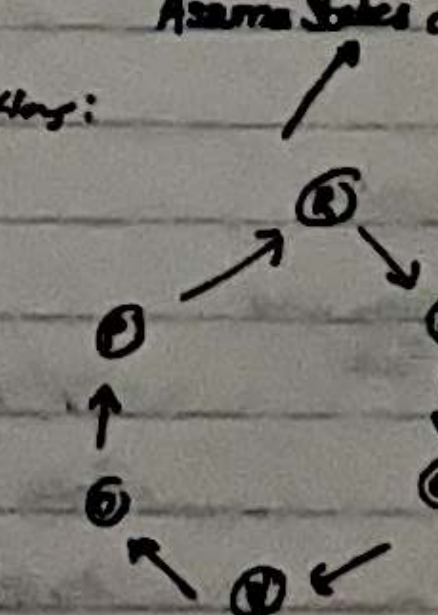
↳ If it is in a state at one instance, we know the next state it's going to be in in the next moment.

Rule of Updating Configurations:

$R \to B$ $\quad G \to O$

$B \to Y$ $\quad O \to P$

$Y \to G$ $\quad P \to R$



Assume states are mutually exclusive, if it is red, it cannot be Green.

We don't know where the system started, but once we make an observation and know the rule, we can make predictions of future states of the system.

We no need to assume the cube has no geometric symmetry } → $P(i) = 1/6$

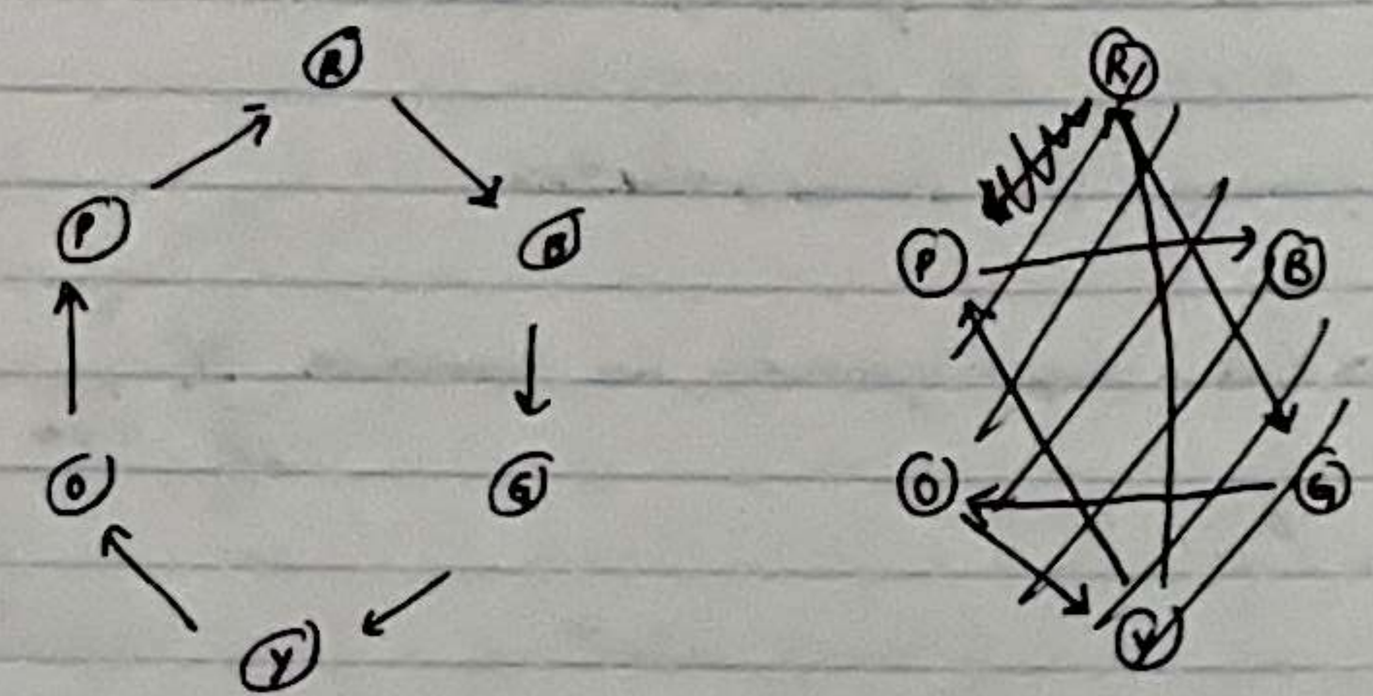
↳ Time spent in each state } Got this w/o knowing the structure of the system.

1/6 of a color being observed

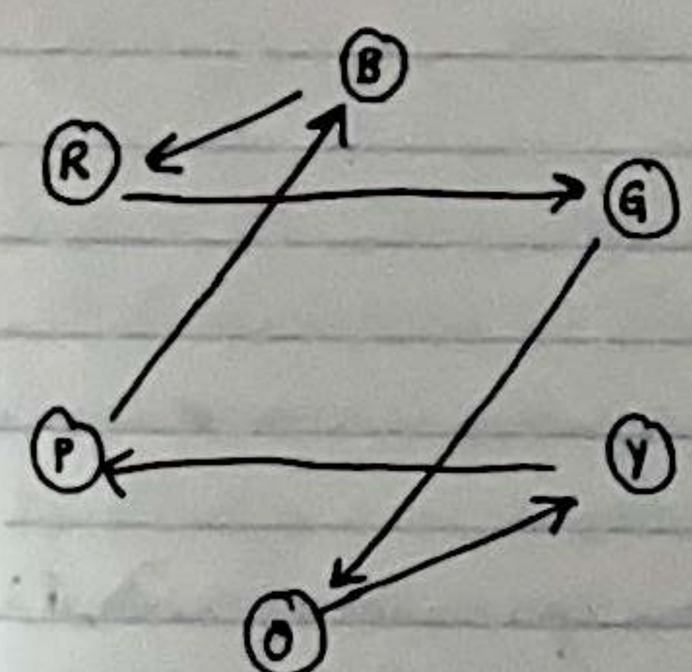
We Made a prediction w/o knowing the initial conditions of the law of physics of the system, but just observing the system.

All the following systems ~~yield~~ yield the probability of observing a Color as $1/6$.

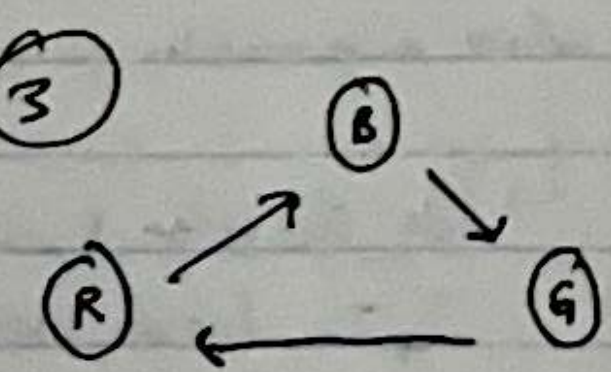
(1)



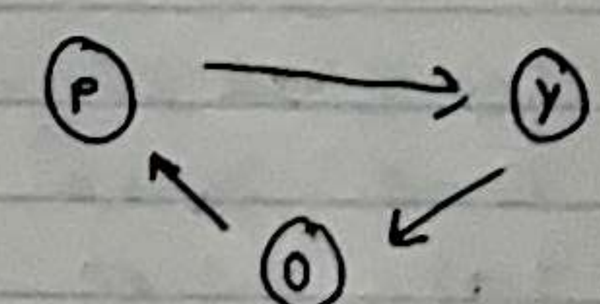
(2)



(3)



+1



−1

All form closed cycles
All have one arrow coming and another coming out of them.

Might not know where started but observed that in either two state of the system, they cycle through only 3 colours.

If on one of these two cycles, we stay on that cycle
If start on upper cycle,
$P(R) = P(B) = P(G) = 1/3$
$P(Y) = 0$

$P(+1)$ or $P(-1)$ → Probability of individual. Cycle.

$P(B) = P(+1)\,P(\text{Blue in cycle } {+1}) = P(R,B,G)$

$\frac{1}{3}$

$P(-1)\ 1/3 = P(Y,P,O)$

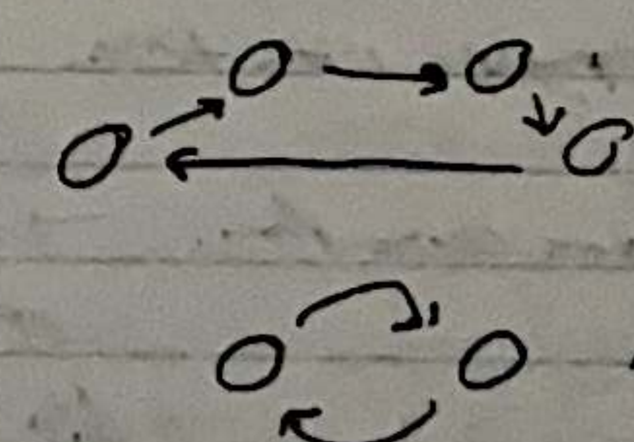
There is a conservation law: The law is the conservation of the number (+1 or −1) configuration.
e.g. Energy.

+1 is the conserved property between these

Energy of Configuration (R, B, G) could be +1
Energy of Configuration (Y, P, O) Could be −1

It keeps us on the same cycle.

Conservation laws Cause the Space of Possibilities to ~~be~~ be divided into cycles or such. } Partition



Also valid

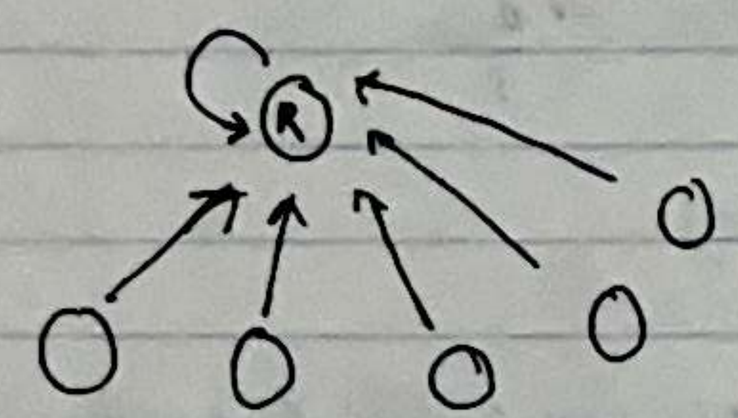
We still need info on the probability of being in each cycle.

How long does the System spend in a cycle.

Some a priori probability telling us the probability of each conserved quantity & the probability when cycling through each state ← That determines probability in statistical Mechanics. → Means System and need.

Laws Violating Conservation of information.

→ Can keep track of what's going forward or backward.



This law is not reversible.
Can go from blue to red but not from red to blue.
Can predict future : Red. → Can tell where will be ~~[illegible]~~ but not where came from.

↳ The law lose track where started.

Primitive law.
Information is never lost.
Distinctions between states propagate with time.

Nature allows to reconstruct where came from. → What about Heat Death of the universe?
No distinction from one point to the other?
Systems wanting to reach equilibrium.

Classical Mechanics, Phase Space and Liouville's Theorem.

In Classical mechanics, the State of a System with N dof can be represented by a 2N-dimensional phase space.

Each point corresponds to a State of the system → The configuration of the system.
Coordinates : $q_1, q_2, \cdots, q_N$
momentum : $p_1, p_2, \cdots, p_N$

The evolution of the configuration of the system is captured in the phase space. It has nothing to do with time evolution.
It just shows a landscape of how one variable influence the other.
We can start at any point & follow tracing to determine the next point (for a given time interval).

The evolution of the system is governed by the Hamiltonian (Total Energy Function) of the system.

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad , \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

$$\boxed{\text{Liouville's Theorem : } \frac{d\rho}{dt} = 0}$$

→ Density of phase space.

The density of states in phase ~~space~~ space remains constant as the system ~~[illegible]~~ evolves according to Hamiltonian Mechanics.

Volume of a phase space is conserved under evolution. ←

↳ Divergence of phase space flow = 0

$\nabla \cdot \vec{v} = 0$ ← Phase space velocity.

Shows that evolution in Hamiltonian phase space is incompressible. → Evolution does not create or destroy phase space.

Implications : Predictability → Conservation of phase space volume is important to deterministic Classical mechanics

Allows to transition to Statistical Mechanics.

Reversibility : Equation of motion preserve information about the system's initial state.

Counter Example to Liouville's Theorem.

Friction : Wherever we start, we come to rest.
↳ Same as come to rest wherever start.
↳ Violation of law that distinctions are preserved.

How does friction prevent the volume of a phase space from being conserved?
↳ Reaches a singular state.

As a system experiencing friction evolves, all flow start to point to a singular point (0) → System at rest.

If the distinctions between the states makes the volume of the phase space, with decreasing distinction between states, the volume decreases, violating Liouville's Theorem.

$$\boxed{\frac{d^2 x_n}{dt^2} = -\gamma \frac{dx_n}{dt}}$$

↳ Exponentially come to rest at 0 quickly.

All pointing to rest.

functions of Total Energy. → Conserved

Liouville's Theorem applies Solely to Hamiltonian Dynamics.
When friction/drag acts, energy dissipates and cluster towards attractors (rest)
The phase space shrinks because energy is dissipated from the system.
The phase volume can be preserved by adding extra thermal dimensions to the system.

Viscous drag is a valid physical law, but information is lost from the system as it comes to rest.
↳ Room quickly cools down
Closed System (Weird) → Things getting to rest quickly

Simpler / Easier to describe.

2nd Law Says things get more complicated.

Conservation of info : Every State must have 1 arrow in & 1 arrow out.
↳ No end state

Over time no. States with non-zero probability remain constant and $P_r = \frac{1}{M}$

Property : N = Total no. of States
M < N Where M is a Subset of N
$P_r(\text{M-states}) = 1/M$, $P_r(\text{other-states}) = 0$
e.g. $N = \{R, B, G, Y, O, P\}$
$M = \{R, B, G\}$

P_r distribution changes..

* A bad law will see all the probabilities skewed to a singular state after many observations.

Assign probability to only a 3-subset of the set.

We know that from derivation despite not knowing the full laws of physics for the system.

~~Therefore~~: We observe the system for large number of times.
The colors may shuffle but only 3 will appear in one ~~obs~~ experimental observation e.g. $\{R, B, G\}$ or $\{R, Y, O\}$

* Good Law

with $\boxed{P_r(\text{State}) = 1/M = 1/3}$ → Conservation of information
↳ Law persists in all observations and after many observations

Where other states, $P_r = 0$

3-States $P_r = 1/3$
3-States $P_r = 0$

1 experiment, see cycling between 3 states (R, B, G)

other experiment, same cycling between 3 states (R, Y, O)

Good conservation law : Pr distribution for elements of Subset M (of total state N) remains constant

$Pr(M_i) = 1/M$

Bad Law : Start with a Pr distribution and see it evolve to a single state over time.

Quantifying information Conservation.

Under assumption of equal probabilities, let M be the no of occupied states with non-zero, equal probabilities,

M characterize ignorance, if $M = N$, then equal probability for everything, Maximum ignorance.

if $M = \frac{1}{2}N$, then know that system is in one of two states

Minimum ignorance, $M=1$, know precisely that M is in one particular state.

$$S = \log M$$

Entropy

No of states that have appreciably equal probabilities

Could be any one of the elements of M with probability of each state being $1/M$

Entropy is conserved across experiments, colors may reshuffle but probability distribution remains constant...

Conservation of entropy if we can follow the system in details.

In reality our measuring system might fail to keep track of stuff. We might start with lot of knowledge & end with little. *

S increase

* This lost of information is not inherent to the physical equations but because the observer was not careful.

Maximum Value for S, maximum ignorance, when $M = N$ So, $\log N$

M could be infinite

N is usually finite

Maximum entropy is a measure of the total number of states that exist

Entropy is not just the property of a system. It is the property of a system and the observer's state of knowledge.

Continuous Mechanics

Particles moving around with continuous posn & velocity.

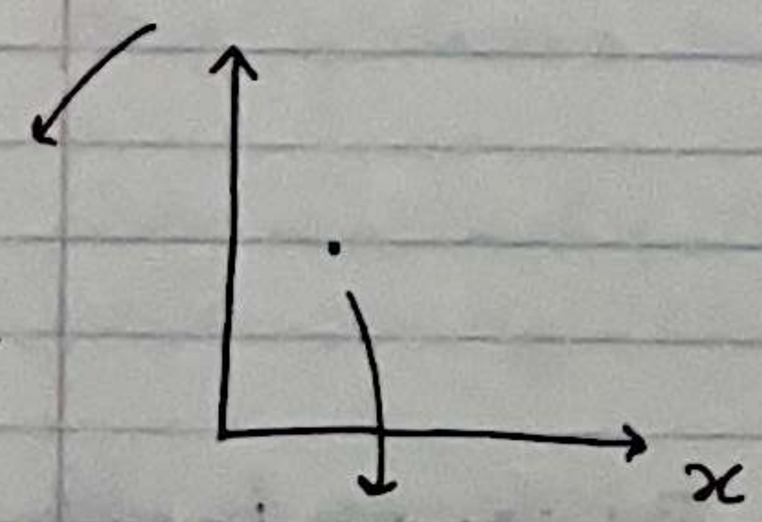
Description of the Space of states of such a mechanical System is by points in a <u>phase space</u>.

↳ Positions (x) & <u>momenta</u> (p)

↳ mass × <u>velocity</u>

p (axis for all momentum dof)

If 10 particles, then 10 p axes

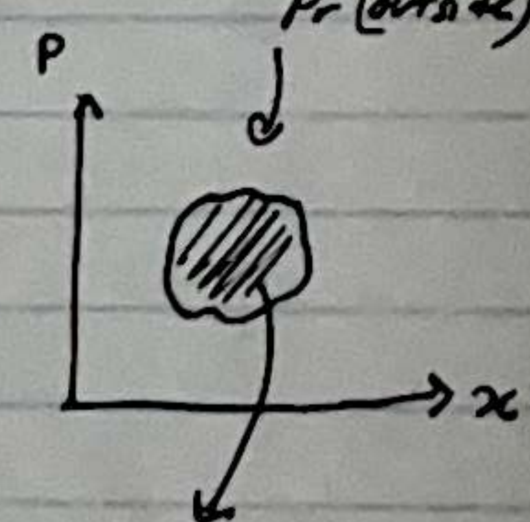


A point represents the possible state of a system, if we know p, x we can get ~~the info~~ the ~~p~~ behavior/state of that system.

Analog of <u>discrete Situation entropy</u> in <u>Continuous Mechanics</u>.

$$\begin{cases} N = \text{Tot.} \\ M < N \\ P_r = 1/M \end{cases}$$ (for all equiprobable states of the subset).

Pr (outside) = 0

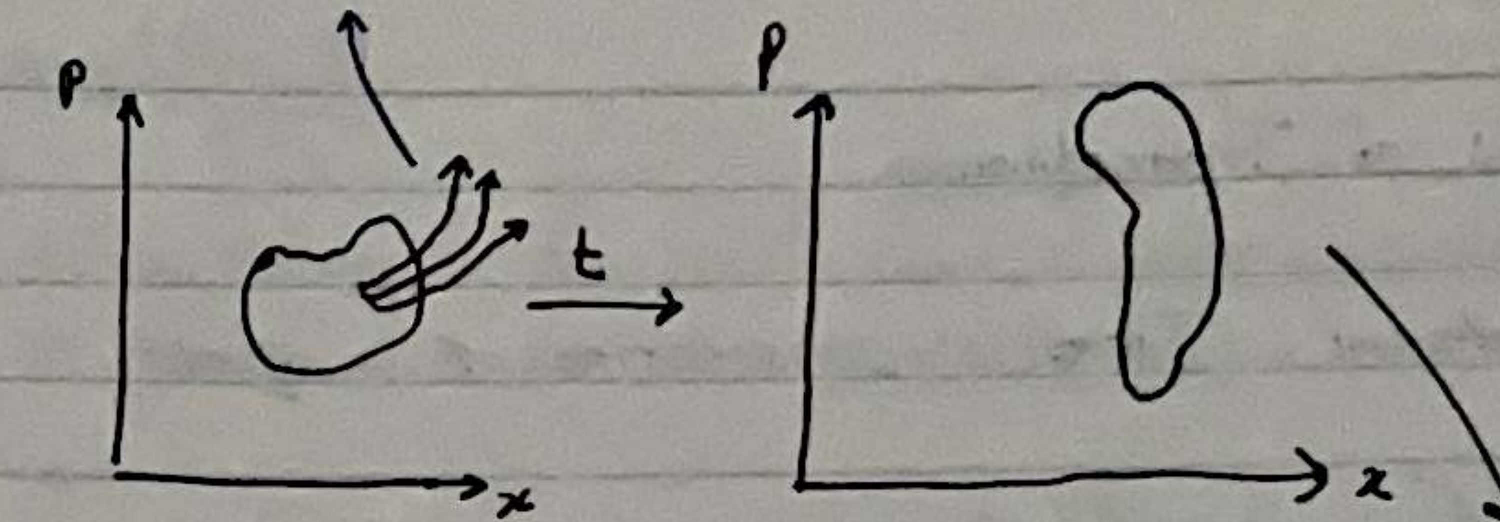


Pr (in the state) Subset = $1/m$.

As time evolves, the system changes from one State to the other, e.g. As the journey goes on, the car accelerates to go up ~~the~~ a hill and decelerates when going down, States change.

This represents moving from one point (state) to another in the phase space.

Points move like fluid flow.



~~Also~~ M states volume after some time.

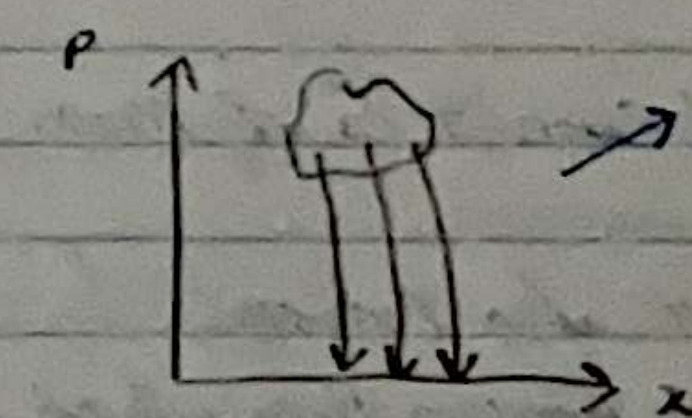
Liouville's Theorem Say that that volume is preserved.

~~If~~ If Start with M number state & follow equations of motions, Will come back to M-number of states ~~(...)~~

↳ (Same number but different states)

↳ Start with uniform Pr distribution, end with uniform Pr distribution.

If consider friction case → P become 0.



→ Region squeezed to 1 dimension (No Volume)

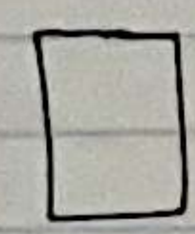
↳ LT prevents that by saying if the region is squeezed in one dimension, it <u>must expand</u> in <u>another</u>.

↳ Pr distribution is spread out in <u>other component</u> of the <u>phase space</u>.

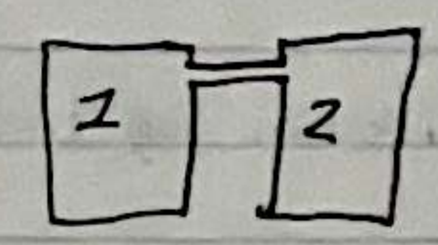
1st Law of Thermodynamics.

↳ Statement : There is conservation of a quantity (Energy)

$$\frac{dE}{dt} = 0 \Big\} \text{ Energy conservation in closed System}$$



If 2 systems connected,



$$\boxed{\frac{dE_1}{dt} = -\frac{dE_2}{dt}}$$

$$\frac{dE_1}{dt} + \frac{dE_2}{dt} = 0$$

If lose energy on one side, gain it on the other.

Assumption : If System is made of 2 parts, the energy is the sum of the 2 parts.

↳ e.g. E_k of Earth + E_k of Moon = Total energy of each particle

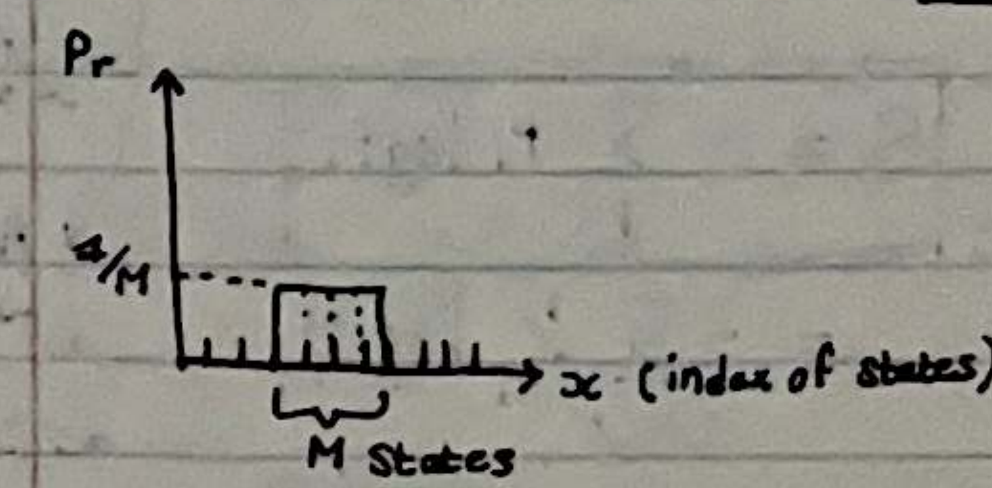
↳ But also Potential energy between them which belong to both of them.

↳ Cannot divide system into two parts.

Interaction of systems can be negligible compared to energy of System.

Entropy (Part II)

We defined entropy for those specific cases of uniform distribution, $P_r = \frac{1}{M}$ and $P_r = 0$ elsewhere. This was also a discrete distribution.



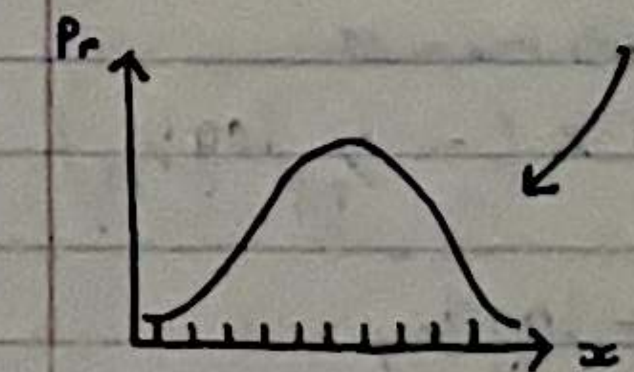
We defined $S = \log M$

↳ Measures ignorance in the distribution.

~~The subset M could be any of the indices with same P_r.~~

The observation could be any indices in the subset M.

But, in reality, we usually have continuous distributions with non-uniform probabilities.



We need another more general def$_n$ of entropy.

Shannon Entropy

$$S = -\sum_i P_i \log P_i$$

The entropy of a distribution is the average* no of states which are importantly contained in that distribution.

Narrower - Small S

Broader - larger S

* The entropy is the expected value of the Self-information or Surprisal. It quantifies how surprising a random variable is on average.

Self-Information $\boxed{I \overset{\text{def}}{=} -\log_b[Pr(x)] = -\log_b(P)}$

$$S = \text{Exp}[I] = -\sum_i P_i \log P_i \qquad -\log P_i$$

↳ Weighing each ~~log P_i~~ for expected value

P goes to 0 faster than log p can go to infinity.

Now, $\boxed{\lim_{P\to 0} P\log P = 0}$

$\lim_{P\to 0} P\ln P$: let $P = e^{-x}$

$\Rightarrow e^{-x} \ln e^{-x}$

$= -x e^{-x}$

~~$\Rightarrow P \to 0, \ln P \to -\infty$~~

~~$\ln P = \ln e^{-x}$~~

~~$\ln P = -x$~~

~~$x = -\ln P$~~

As $P \to 0$

$e^{-x} \to 0$

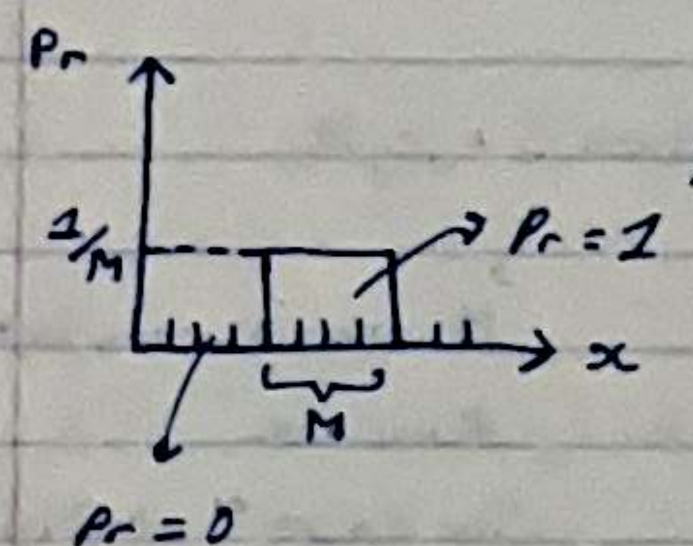
$\Rightarrow x \to \infty$

$\Rightarrow \lim_{x\to\infty} -x e^{-x}$

LR $\to \lim_{x\to\infty} \frac{-x}{e^{x}}$

$= \lim_{x\to\infty} -\frac{1}{e^{x}} = 0$

$\Rightarrow \lim_{P\to 0} P\log P = 0$



$\frac{1}{M} \times M$

$$\boxed{S = -\sum_i P_i \log P_i}$$

$$= -\sum \frac{1}{M} \log \frac{1}{M}$$

P_i is uniform

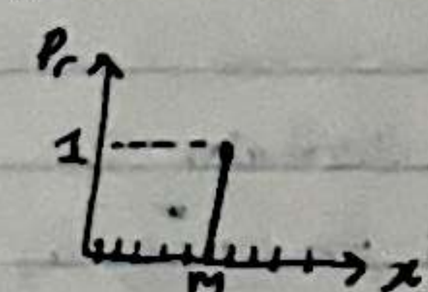
We have M total indices with value M

$$= \cancel{M} \times \left(-\frac{1}{\cancel{M}} \log \frac{1}{M}\right)$$

$$= -\log \frac{1}{M}$$

$$= \log M \longleftarrow \text{Same as Previous Case}$$

If we have perfect information, $M = 1$



$M=1$

$S = 0 \to$ Complete knowledge.

More we know, Smaller entropy.

But more we know, more dof in system.

Implies that contributions of states with little probabilities will be also very little

For the uniform distribution, those with significant probabilities all have $Pr = \frac{1}{M}$

Information Theory defn of entropy.

$$\boxed{S = -\sum_i P_i \log P_i}$$

Entropy is the average measure of Surprisal in a ~~System~~ Probability distribution

↳ More noise, More Suprise in Seeing "weird" ~~obser~~ observation means more entropy.

↳ More diversity.

It is not something interest to a system Like energy.

~~Th~~ It has to do with Pr distribution on state of possible states.

Defn to do both with system and observer's knowledge

Examples

n coins

(H) (T) (H) (T) (T) . . .

↳ Complete ignorance.

(Assume) All Probabilities are equal.

What is Pi ? $\frac{1}{2}$ or $\frac{1}{2^n}$

What is average? n or 2 ??

Probability distribution is same for all states.

1) Suppose we have ~~M~~ n coins and each of them could be H or T. What is the entropy associated with this?

We have complete ignorance, Pr distribution is same across N coins

↳ All Pr are equal.

↳ We use ln of no of total possible states

$\to N$

no of coins

$N = 2^n$

Total no of states

$S = -\sum P_i \log P_i$

$\frac{1}{n}$

n

$S = n \log 2$

$= -\log \frac{1}{2} \to$ All have equal odds of being H or T.

Entropy is proportional to no of dof.

A unit of entropy : A bit is a unit of entropy for a system which has 2 states.

↳ 1 bit of entropy for each coin.

Example 2

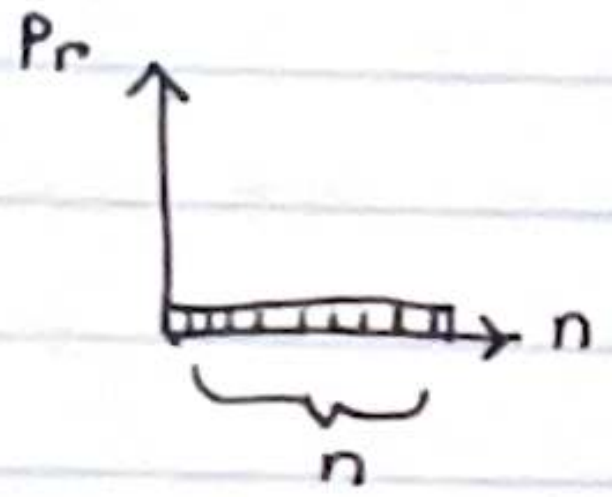
Suppose we have n coins, ~~all are H~~.

~~$\Rightarrow S = 0$~~

$\cancel{S = -\sum_{i}^{n} P_i \log P_i}$ ~~1/n~~

What is the entropy when all coins are H?

$S = 0$. Why?



→ All states have equal probabilities
$P_r = 1$

↳ What is the probability distribution we are talking about?

↳ We have n indices and we are tracking the odds of getting H at each index.

We know that all coins are H.

↳ What am I not getting here?

Our distributions consists of n equiprobable coins