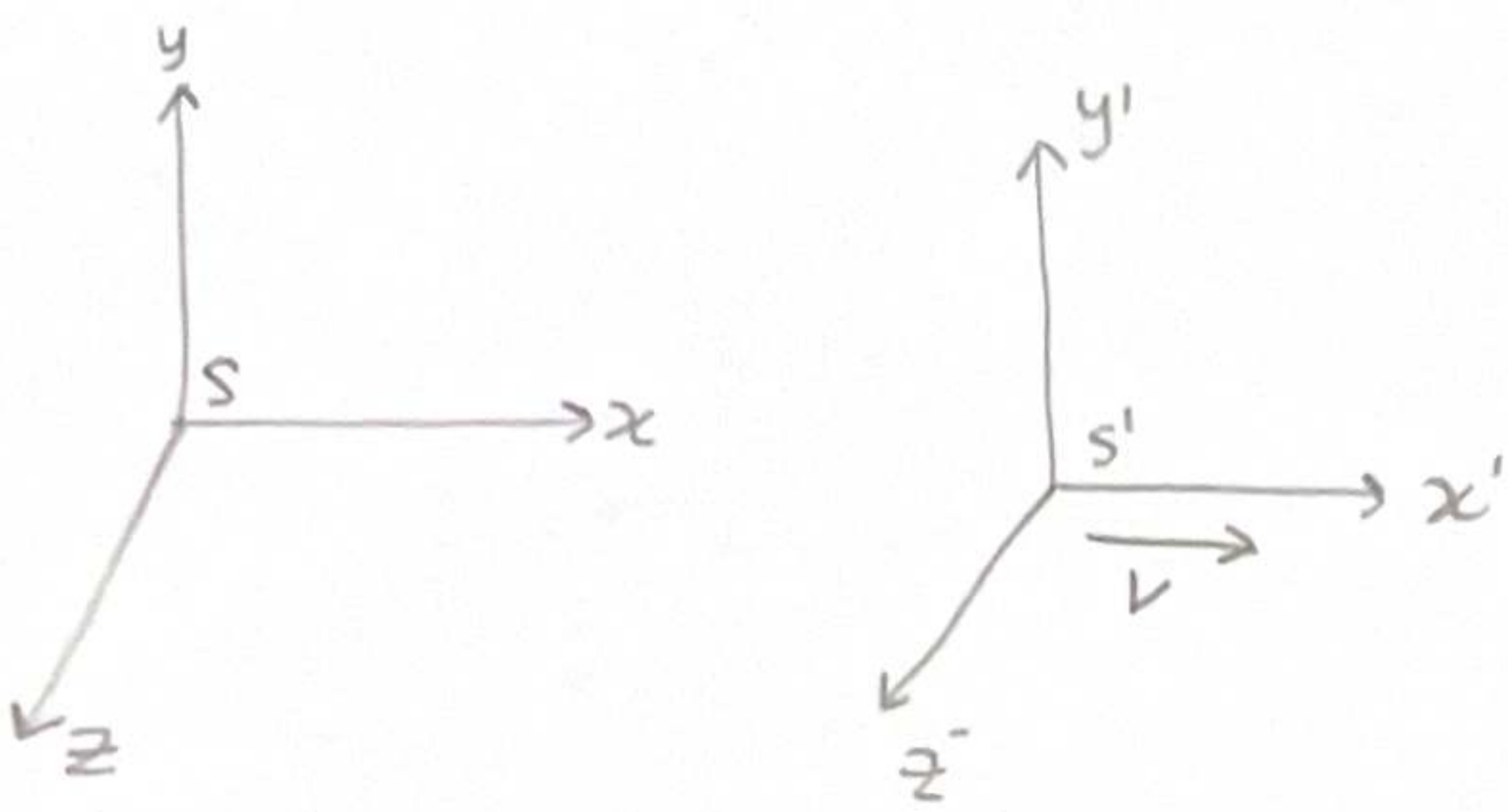
The Galilean Transformation

Suppose there are 2 reference frames designated by S and S' such that the coordinate axes are parallel as shown. S' is moving w.r.t S with velocity v (as measured in S) in the x direction. The clocks of both system were synchronised at $t = 0$ and they move at the same rate.



$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$

Galilean Transformation

Enable us to relate measurement in one frame of reference to another.

Suppose we measure velocity of a vehicle in S and we want to know what would be its velocity in S'

$$V'_x = \frac{dx'}{dt} = \frac{d(x - vt)}{dt} = V_x - V$$

Newton's Second law.

$$f' = m'a' = m'\frac{d^2x'}{dt'^2} = m\frac{d}{dt'}\left(\frac{dx'}{dt'}\right)$$

$$= m\frac{d}{dt}\left(\frac{d(x - vt)}{dt}\right)$$

$$= m\frac{d(V_x - V)}{dt} = m\frac{dV_x}{dt}$$

$$= ma = f$$

$\Rightarrow$ Second law of motion is invariant under the ~~ga~~ Galilean transformation.

Maxwell Equations (3) & (4)

(3) $\nabla \times B = \varepsilon_0 \mu_0 \dfrac{\partial E}{\partial t}$ (4) $\nabla \times E = -\dfrac{\partial B}{\partial t}$

$$\frac{\partial}{\partial t}(\nabla \times B) = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \longrightarrow \text{Time derivative taken}$$

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial B}{\partial t} \longrightarrow \text{curl taken}$$

$$\frac{\partial}{\partial t} \nabla \times B = \nabla \times \frac{\partial B}{\partial t} \longrightarrow \text{Spacial \& time derivatives commute}$$

$$\text{So, } \nabla \times (\nabla \times E) = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

From identity $\nabla \times (\nabla \times E) = \nabla \nabla \cdot E - \nabla^2 E$

$$\Rightarrow \boxed{\nabla^2 E = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}} \rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$$

Analogy to 1D wave equation

which is satisfied by the wave equation

$$y(x,t) = \sin(x - ct)$$

$$\frac{1}{c^2} = \varepsilon_0 \mu_0$$

$$\boxed{c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

- which represents a wave travelling along x axis with velocity c

- Speed specified

- Speed is independent of source and observer