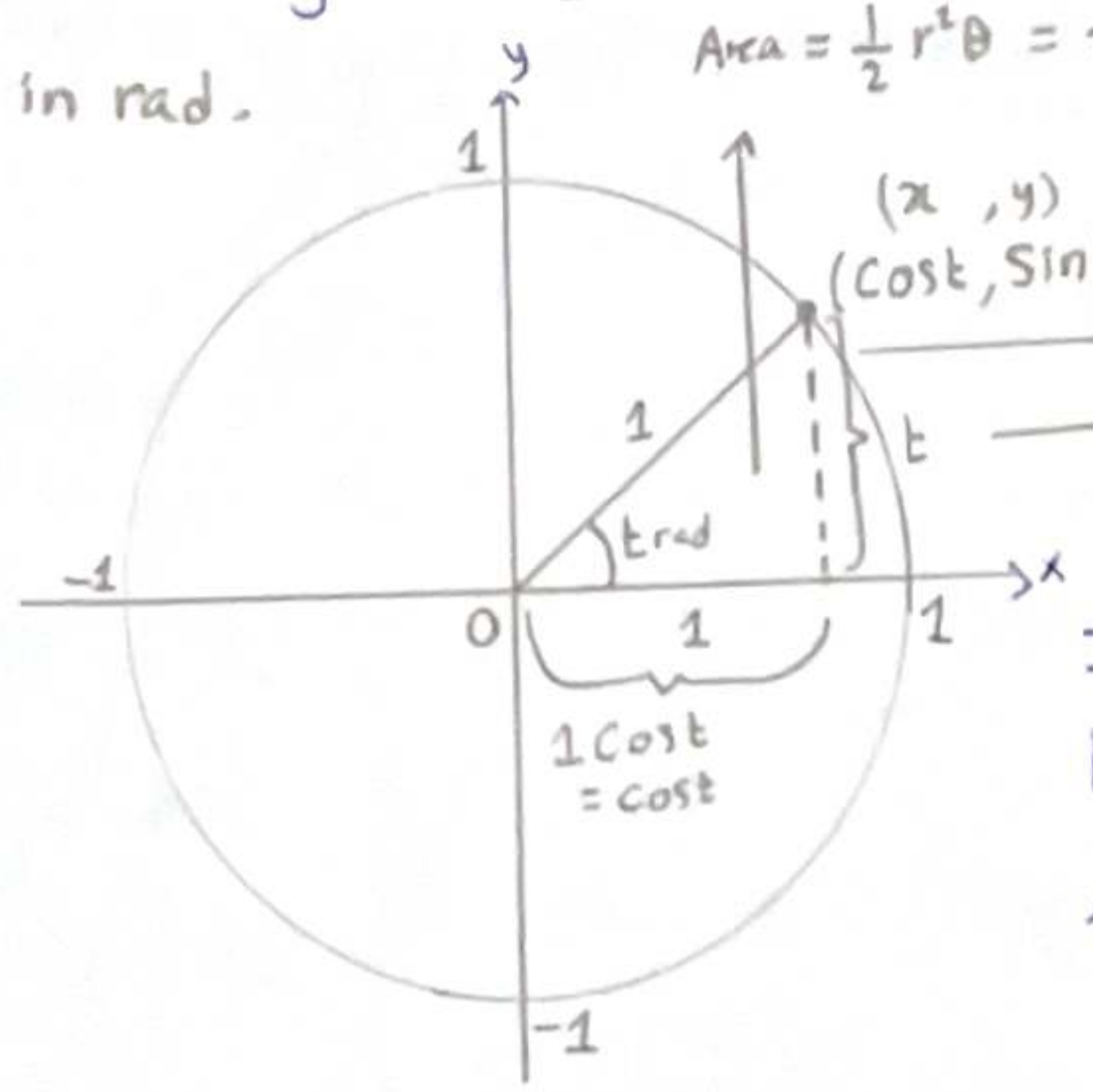
Hyperbolic Trigonometric functions.

Note: unit circle can be used to show all the properties of trigonometric functions.

Circular trigonometric function. (unit circle)

t is in rad.



$\rightarrow 1\sin t = \sin t$

$\rightarrow$ arc length $= r\theta$
$= 1 \times t = t$

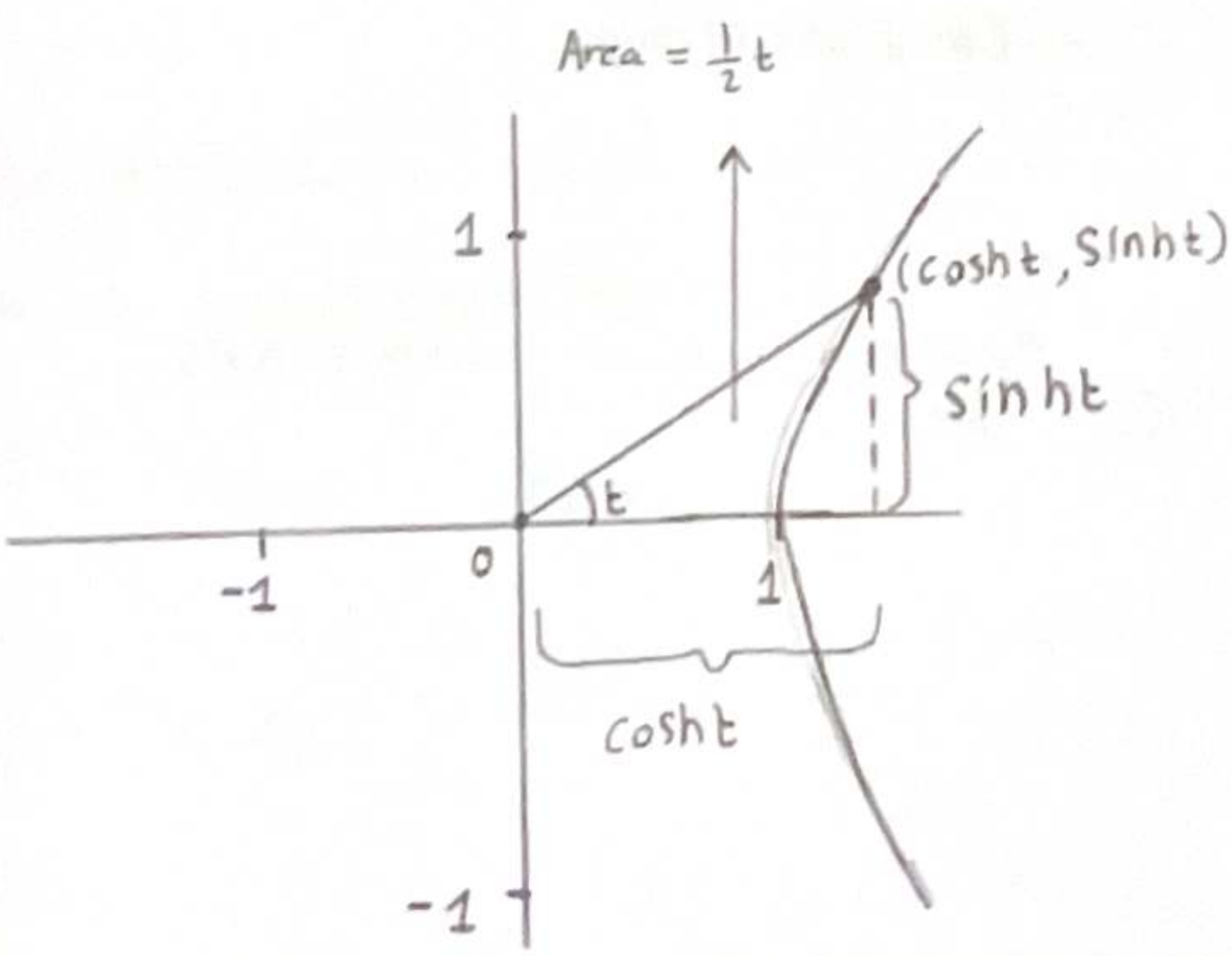
If a point is on arc length t, anticlockwise around the unit circle, from $(1,0)$, then, that point is $(\cos t, \sin t)$

line segment from origin to unit circle sweeps out an area of $\frac{1}{2}t$.

If we use all values of t, the points $(\cos t, \sin t)$ form the circle with equation $x^2 + y^2 = 1$

Unit hyperbola

If a line segment from the origin to the unit hyperbola sweeps out an area of $\frac{1}{2}t$ between $(1,0)$ and a particular point as it moves upwards, then that point is $(\cosh t, \sinh t)$
(x, y)



If we use all values of t, the points $(\cosh t, \sinh t)$ form the right-hand branch of the hyperbola with equation $x^2 - y^2 = 1$

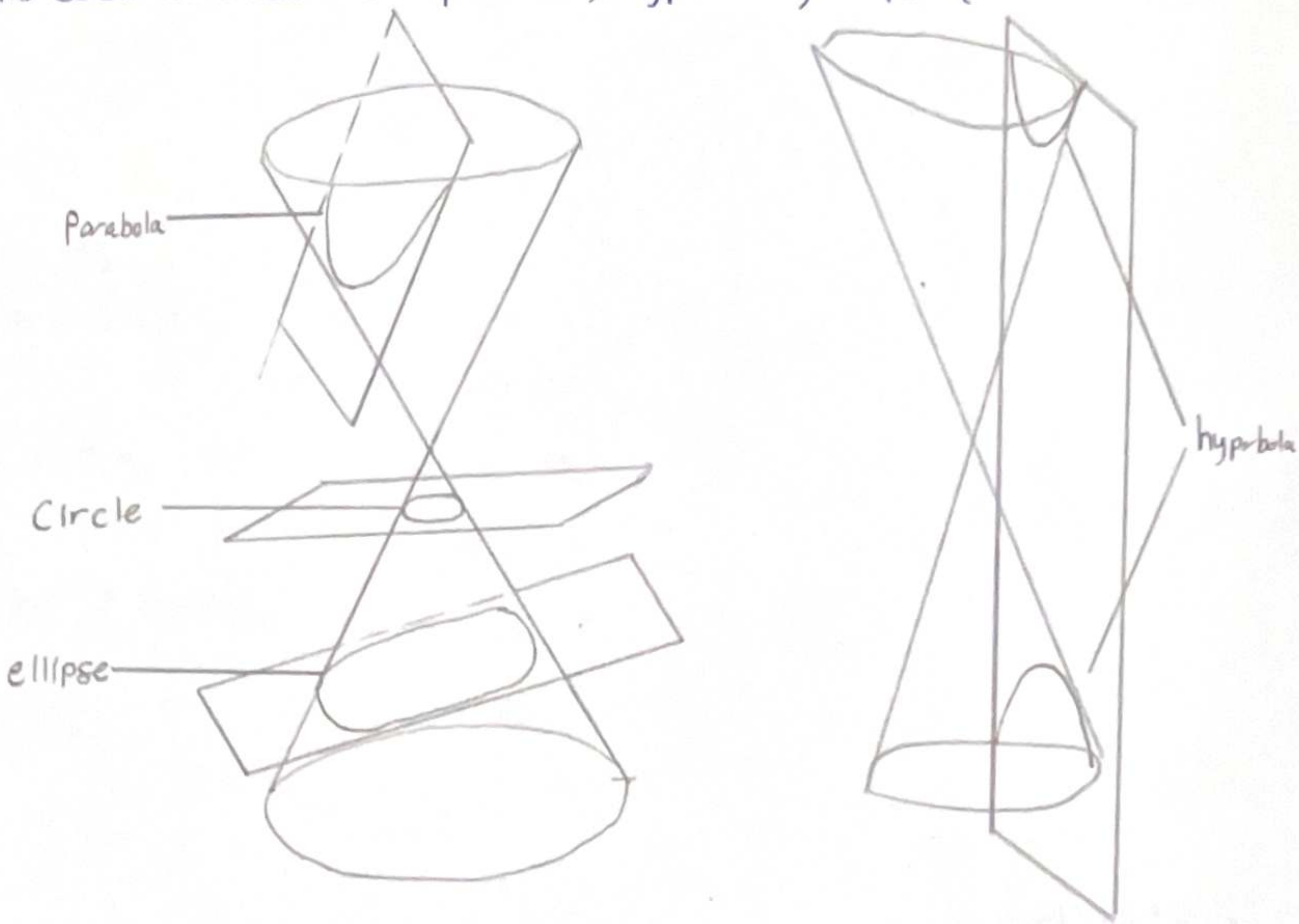
$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \operatorname{cosech} x = \frac{1}{\sinh x}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

Conic Section

A conic Section is a curve obtained as the intersection of the Surface of a cone with a plane. 3 types of conic Section are obtained : Parabola, hyperbola, ellipse (and circle (special type of ellipse)).

To expand on all conic Sections.



Equation of conic Sections.

① Parabola : $y = a(x-h)^2 + k$

Coordinates of maximum value

② Circle : $(x-h)^2 + (y-k)^2 = r^2$ — radius^2

Coordinates of centre

③ ellipse : $\dfrac{(x-h)^2}{a} + \dfrac{(y-k)^2}{b} = 1$

Semi minor axis or Semi major axis.

④ hyperbola : $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

$(\cosh^2 x) - (\sinh^2 x) = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{cosech}^2 x$

$\sinh(2x) = 2\sinh x \cosh x$

$\cosh(2x) = \cosh^2 x + \sinh^2 x$
$= 2\cosh^2 x - 1$
$= 1 + 2\sinh^2 x$

$\cosh(0) = 1 \quad \sinh(0) = 0 \quad \tanh(0) = 1$

$\frac{d}{dx}\cosh x = \sinh x \quad \frac{d}{dx}\sinh x = \cosh x \quad \frac{d}{dx}\tanh(x) = \operatorname{sech}^2 x$

Note $\frac{d}{dx}\cos x = -\sin x \quad \frac{d}{dx}\sin x = \cos x$

Relationship to e^x

Circular trigonometric functions

$e^{ix} = \cos x + i\sin x$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

hyperbolic trigonometric functions

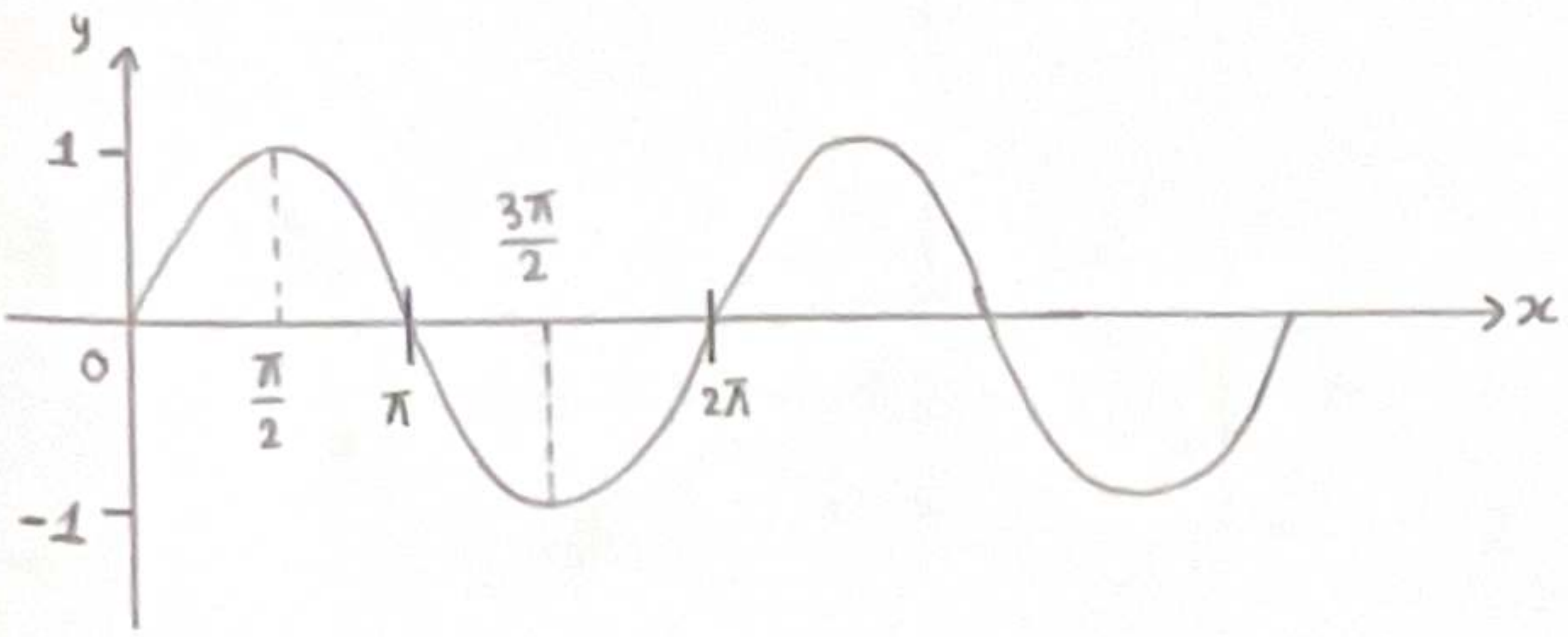
$e^x = \cosh x + \sinh x$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

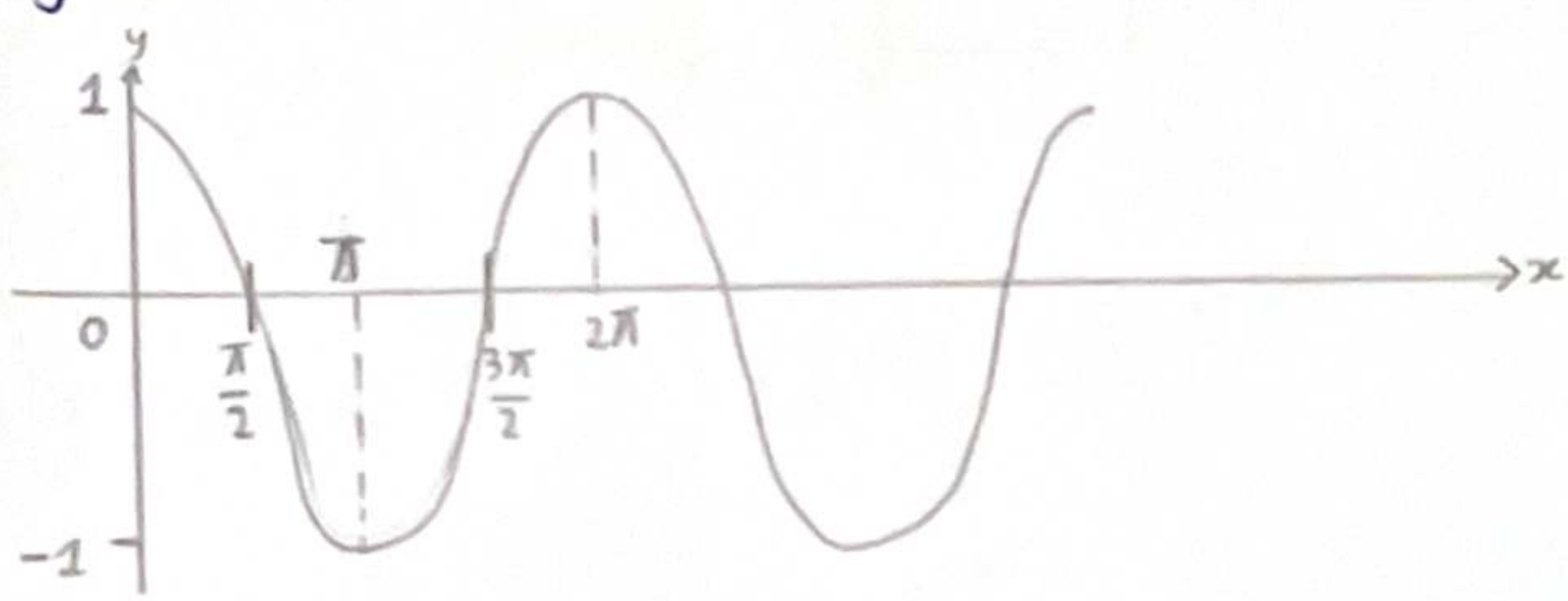
$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Trigonometric Graphs. (Circular)

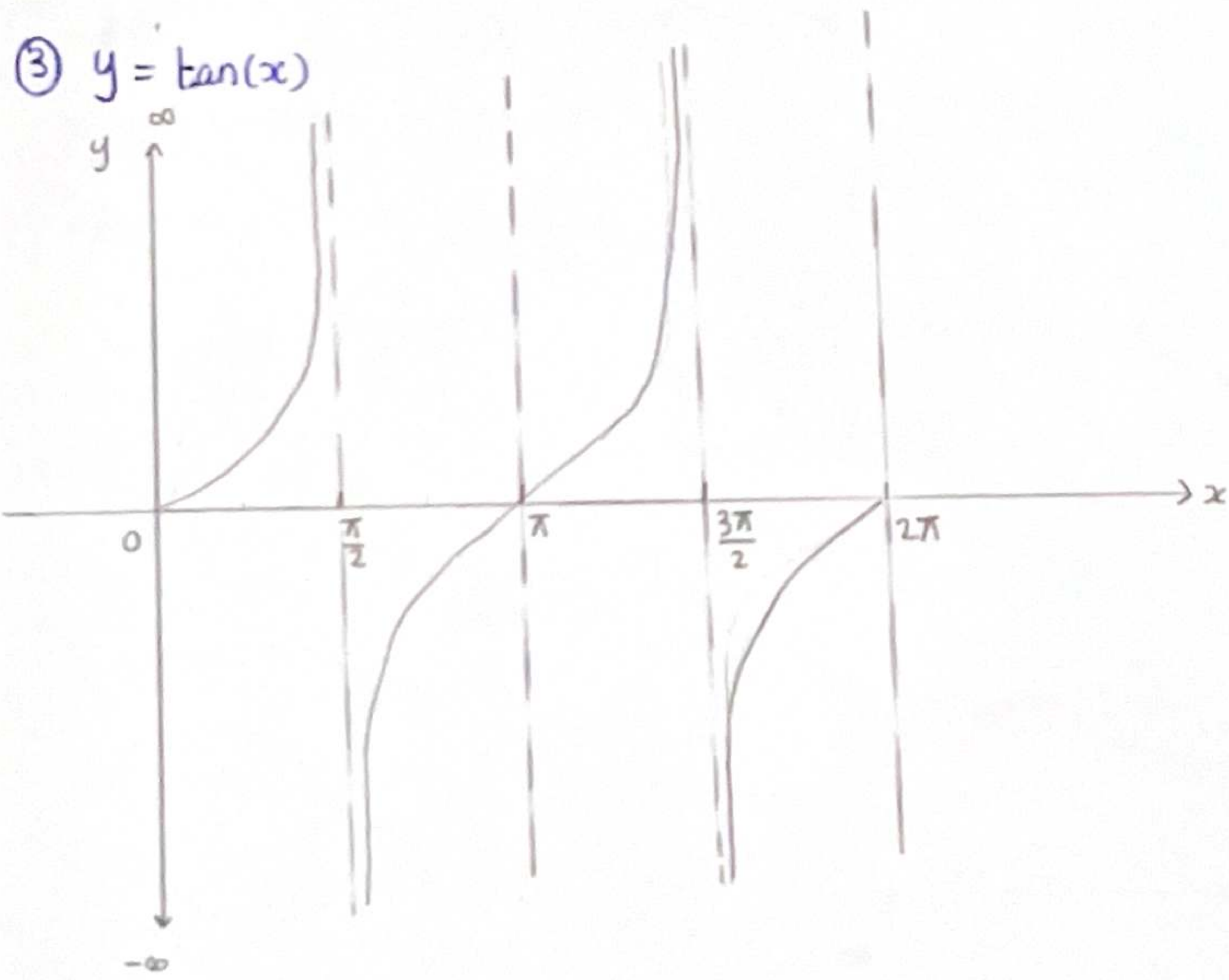
① $y = \sin(x)$ Sinusoidal



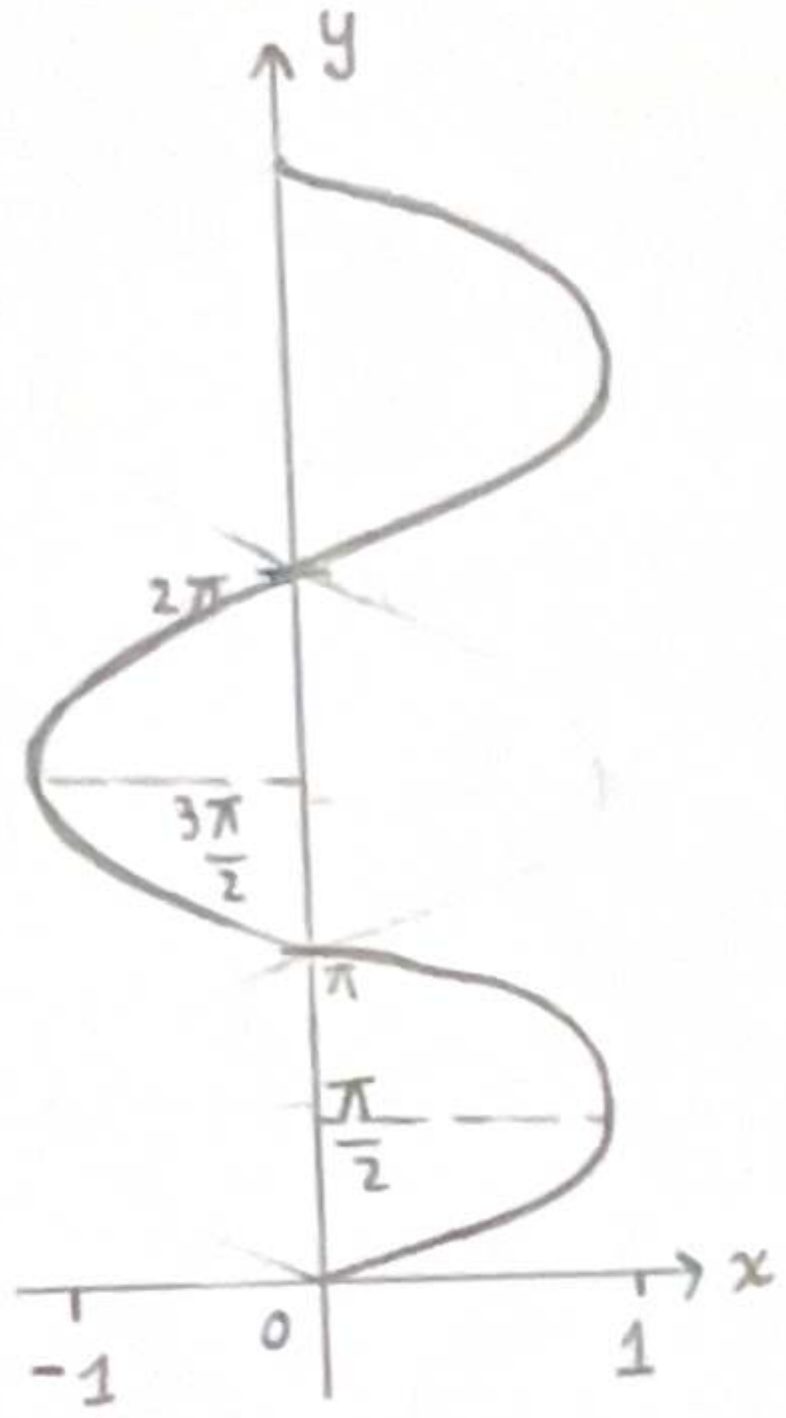
② $y = \cos(x)$ Cosinusoidal



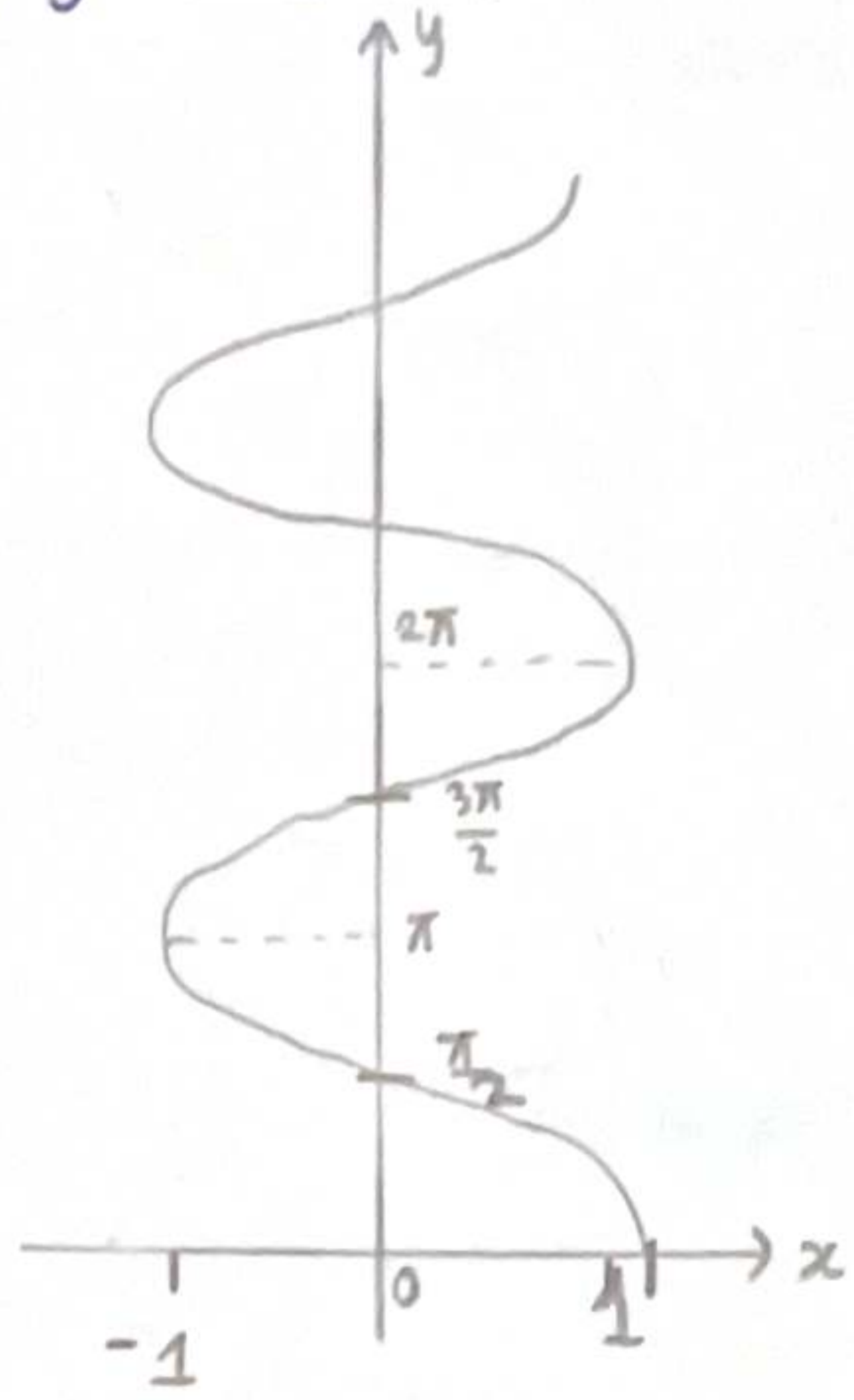
③ $y = \tan(x)$



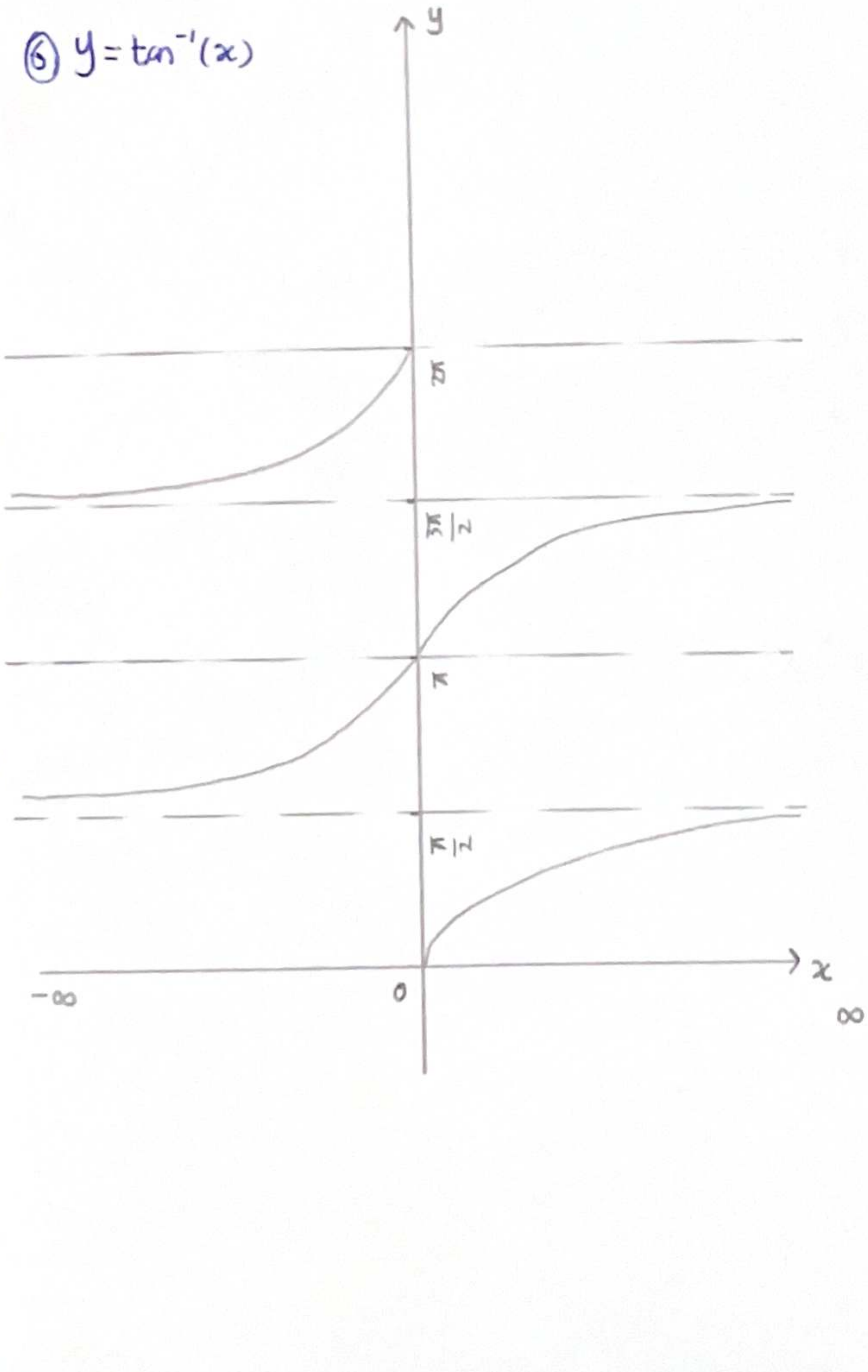
④ $y = \sin^{-1}(x)$



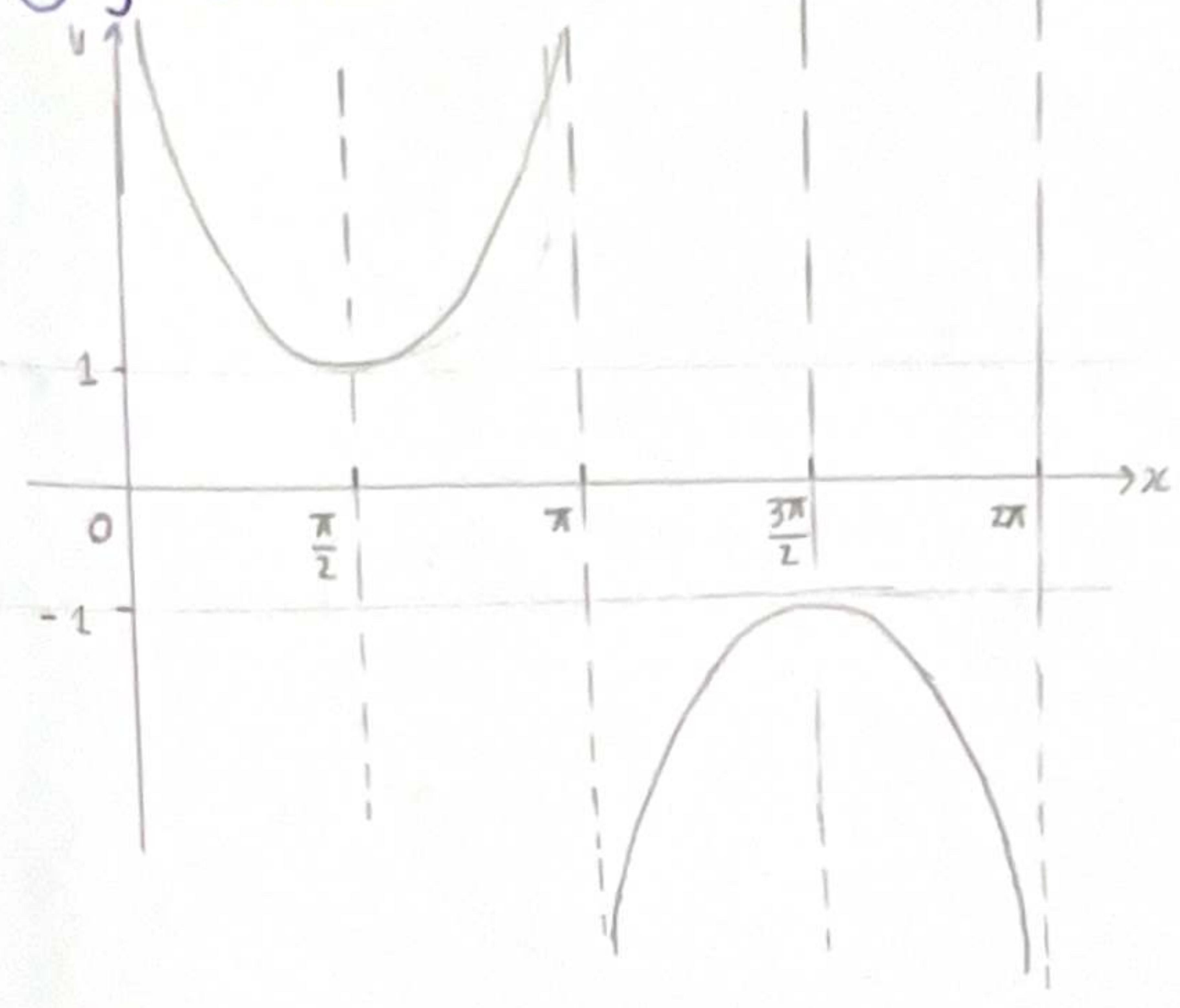
⑤ $y = \cos^{-1}(x)$



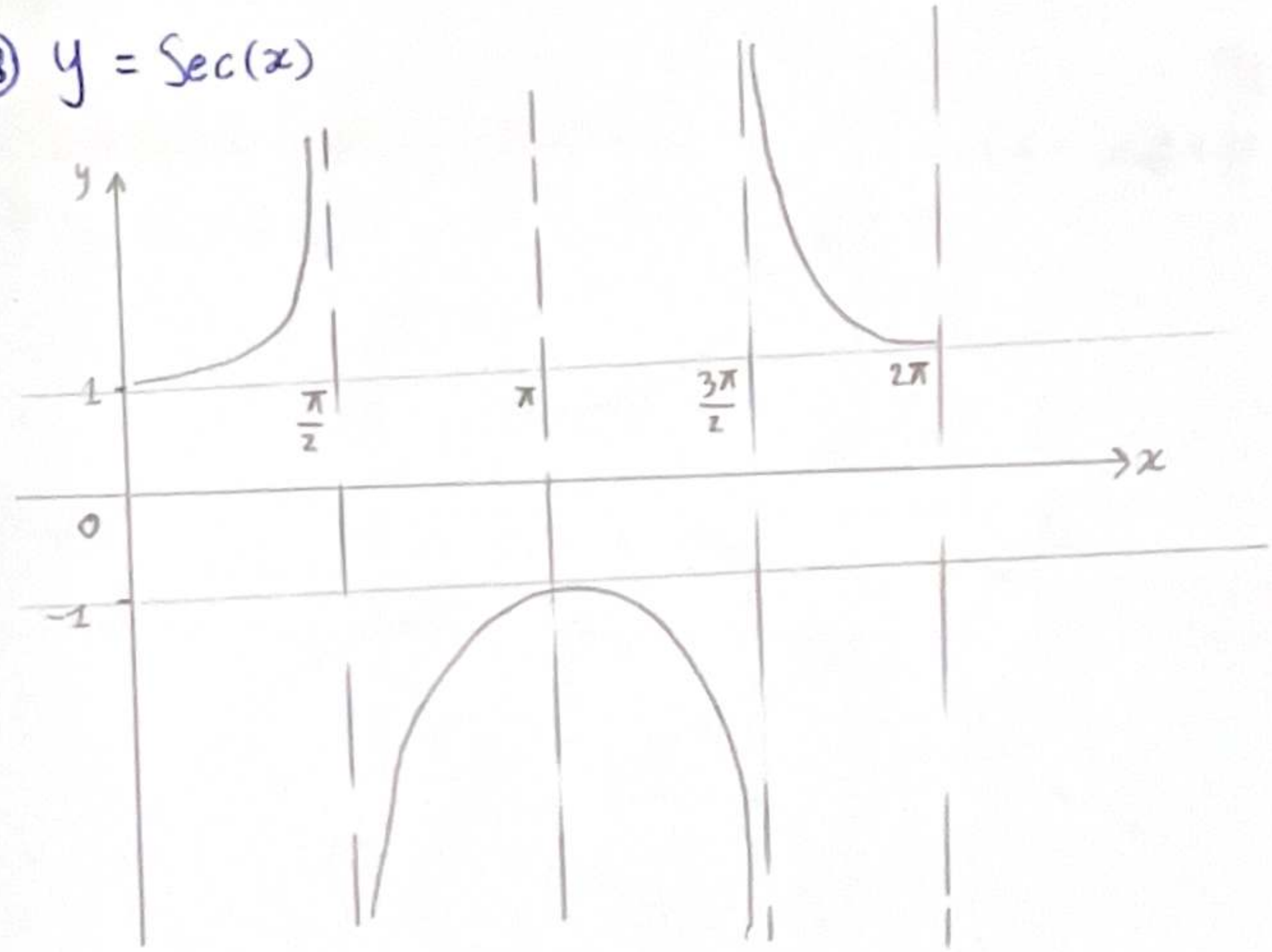
⑥ $y = \tan^{-1}(x)$



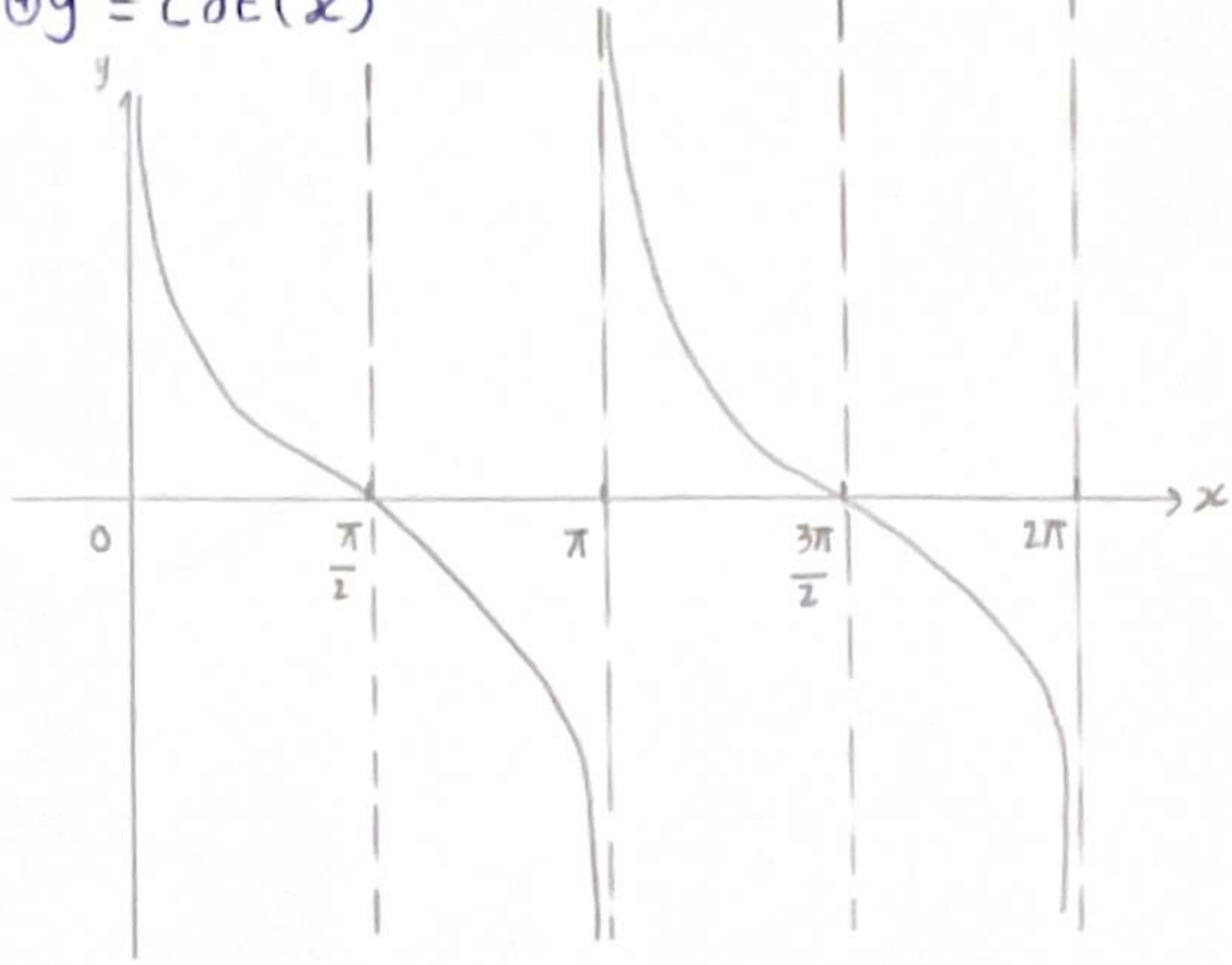
⑦ $y = \csc(x)$



⑧ $y = \sec(x)$

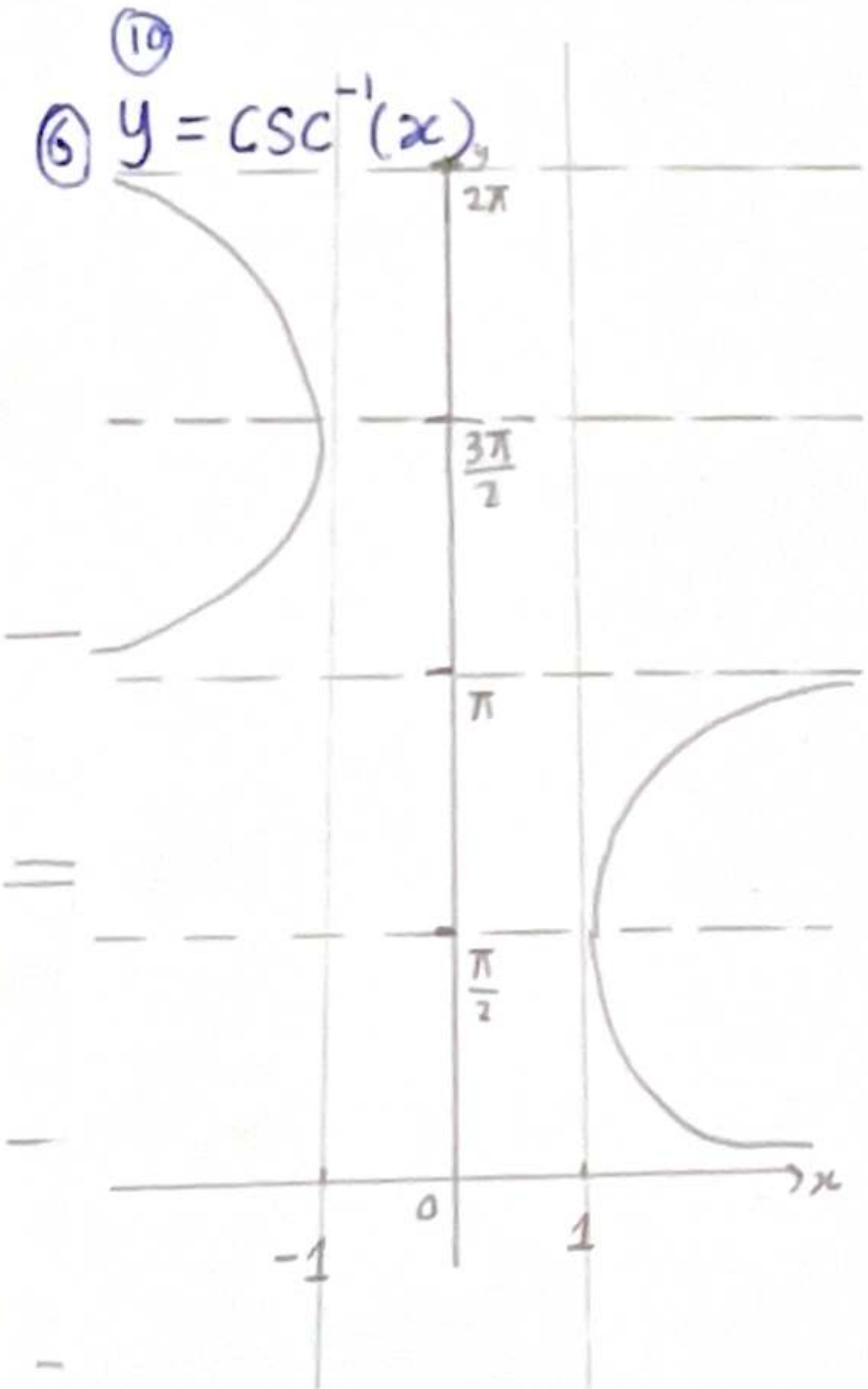


⑨ $y = \cot(x)$



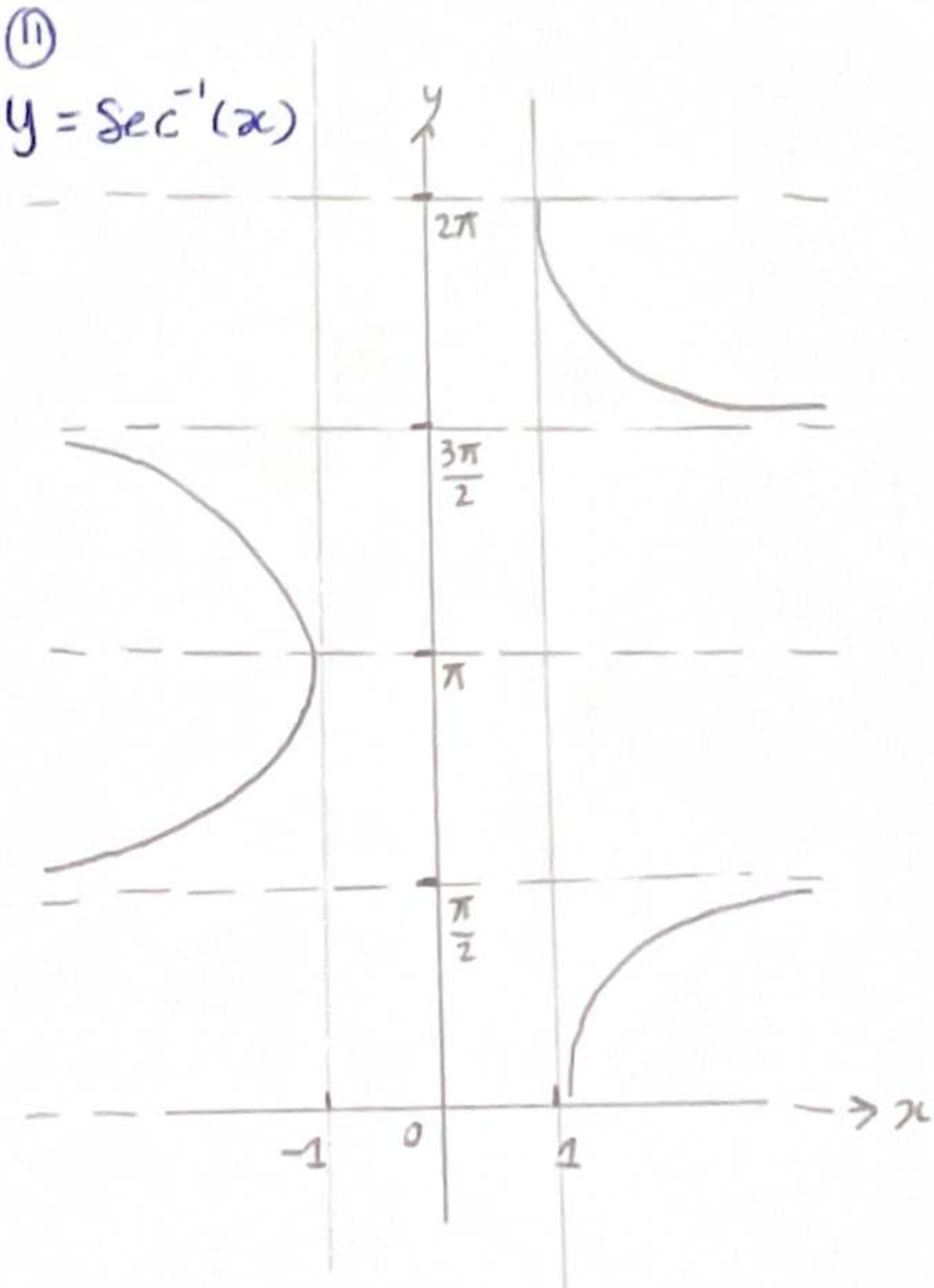
⑩

⑥ $y = \csc^{-1}(x)$

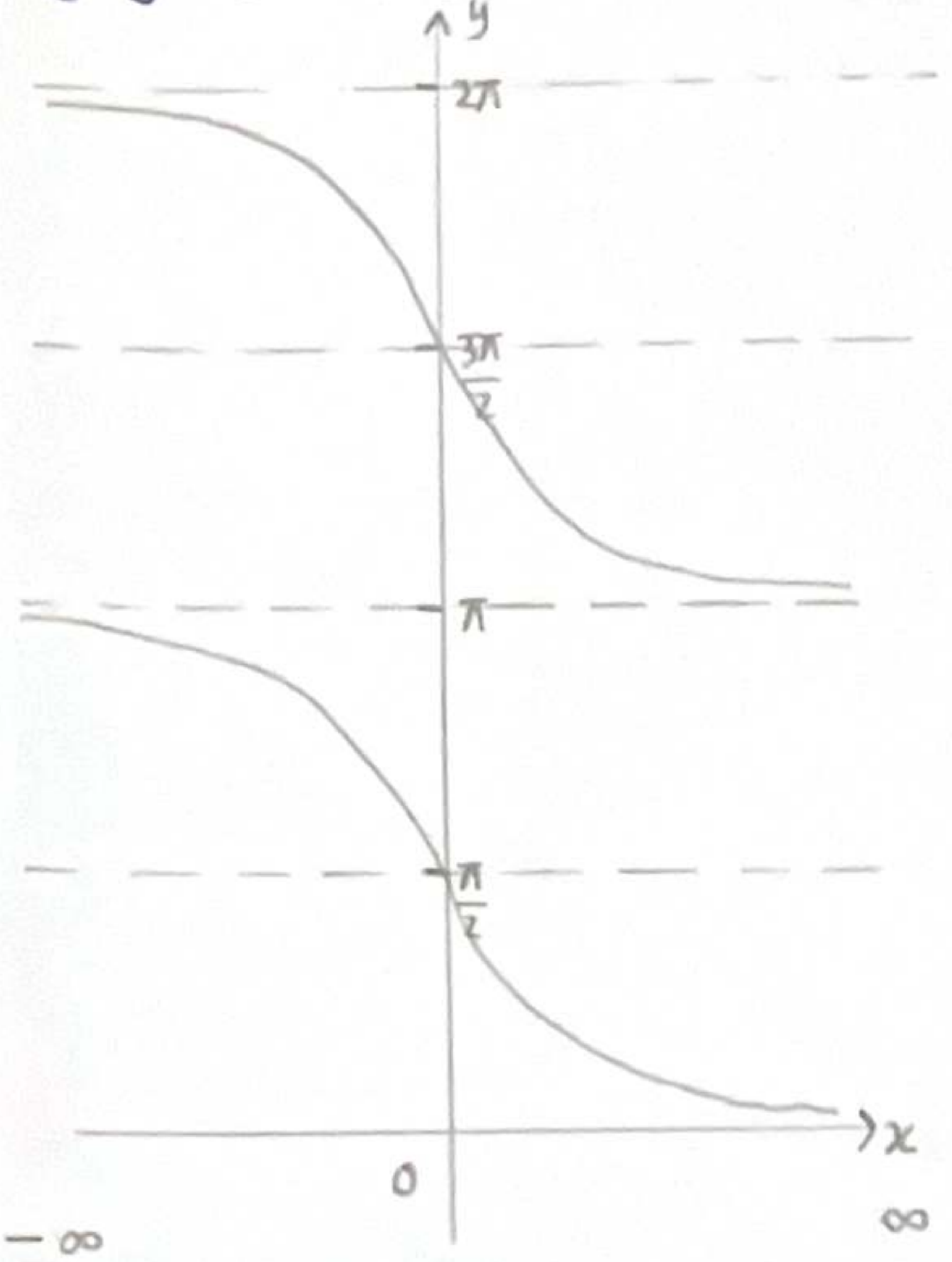


⑪

$y = \sec^{-1}(x)$

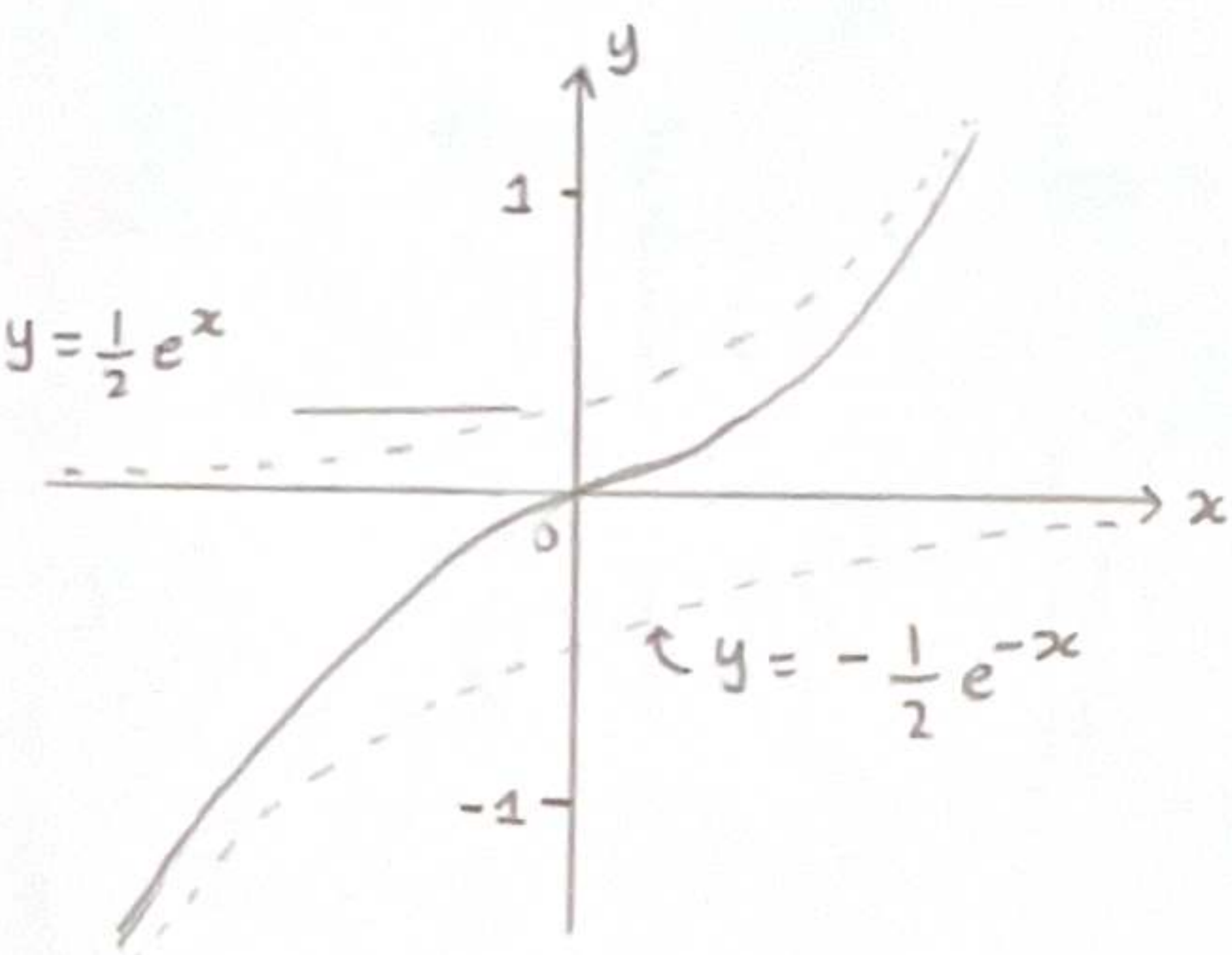


⑫ $y = \cot^{-1}(x)$

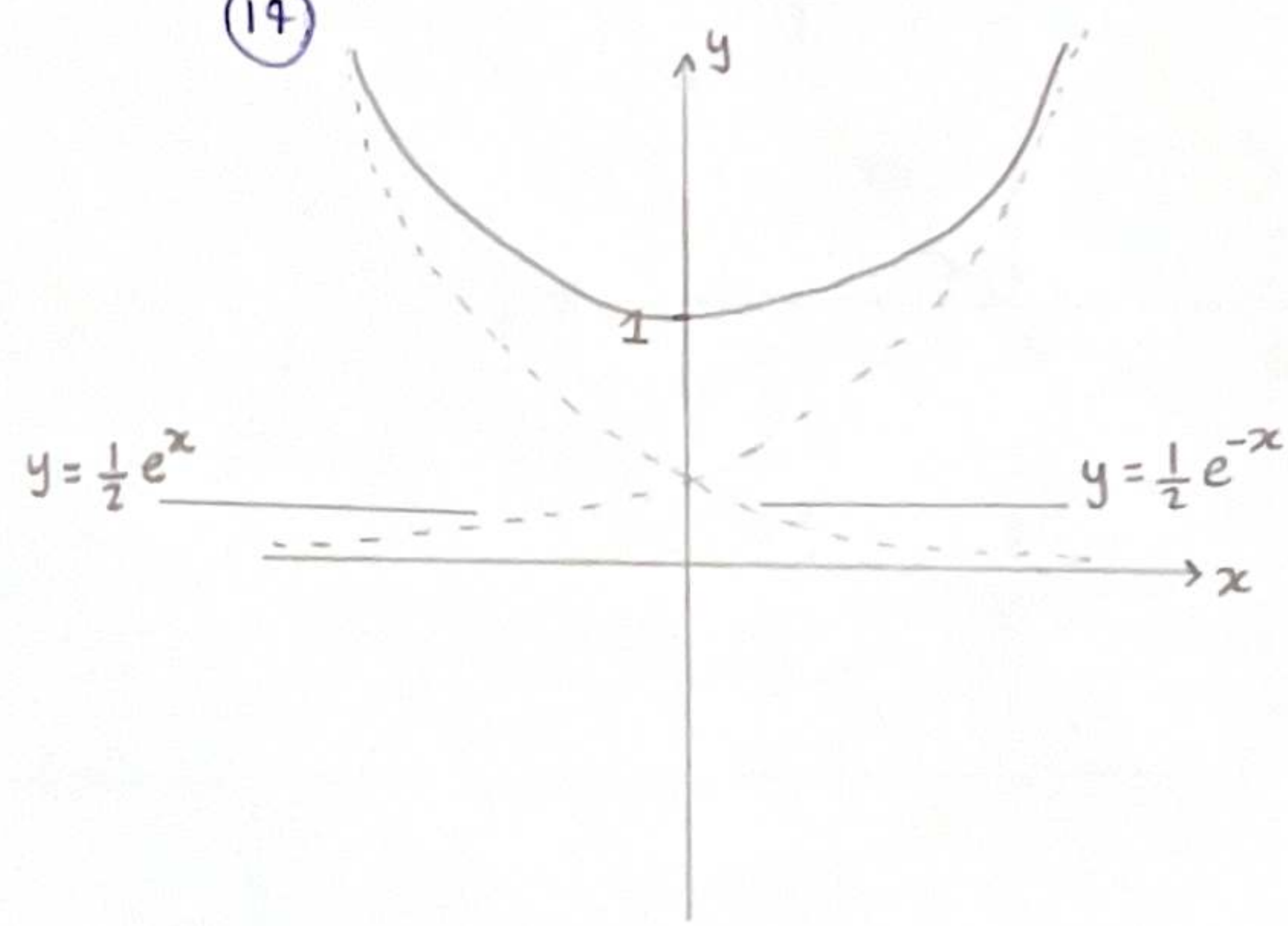


Trigonometric graphs (Hyperbolic)

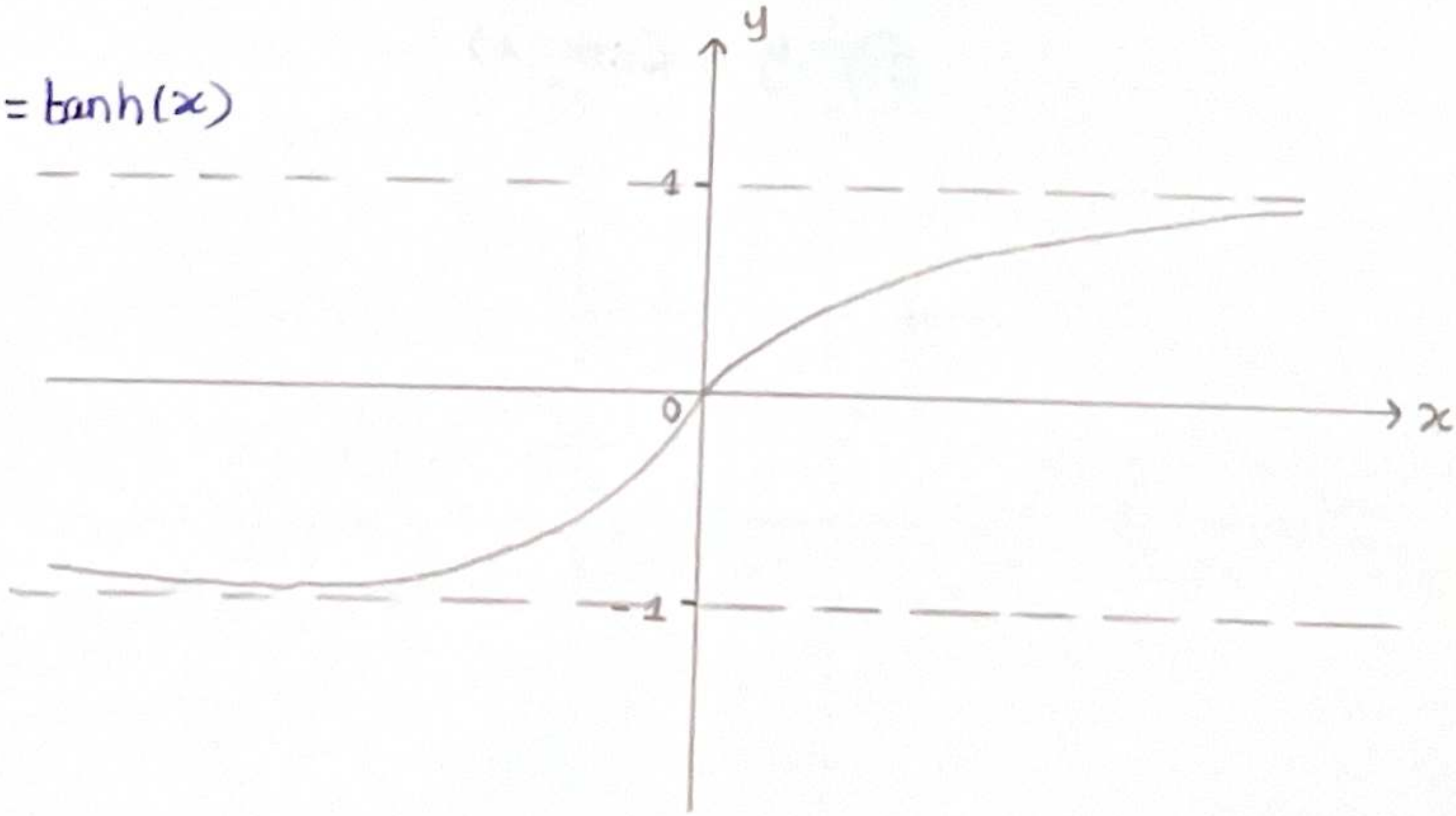
⑬ $y = \sinh(x)$



⑭ $y = \cosh(x)$

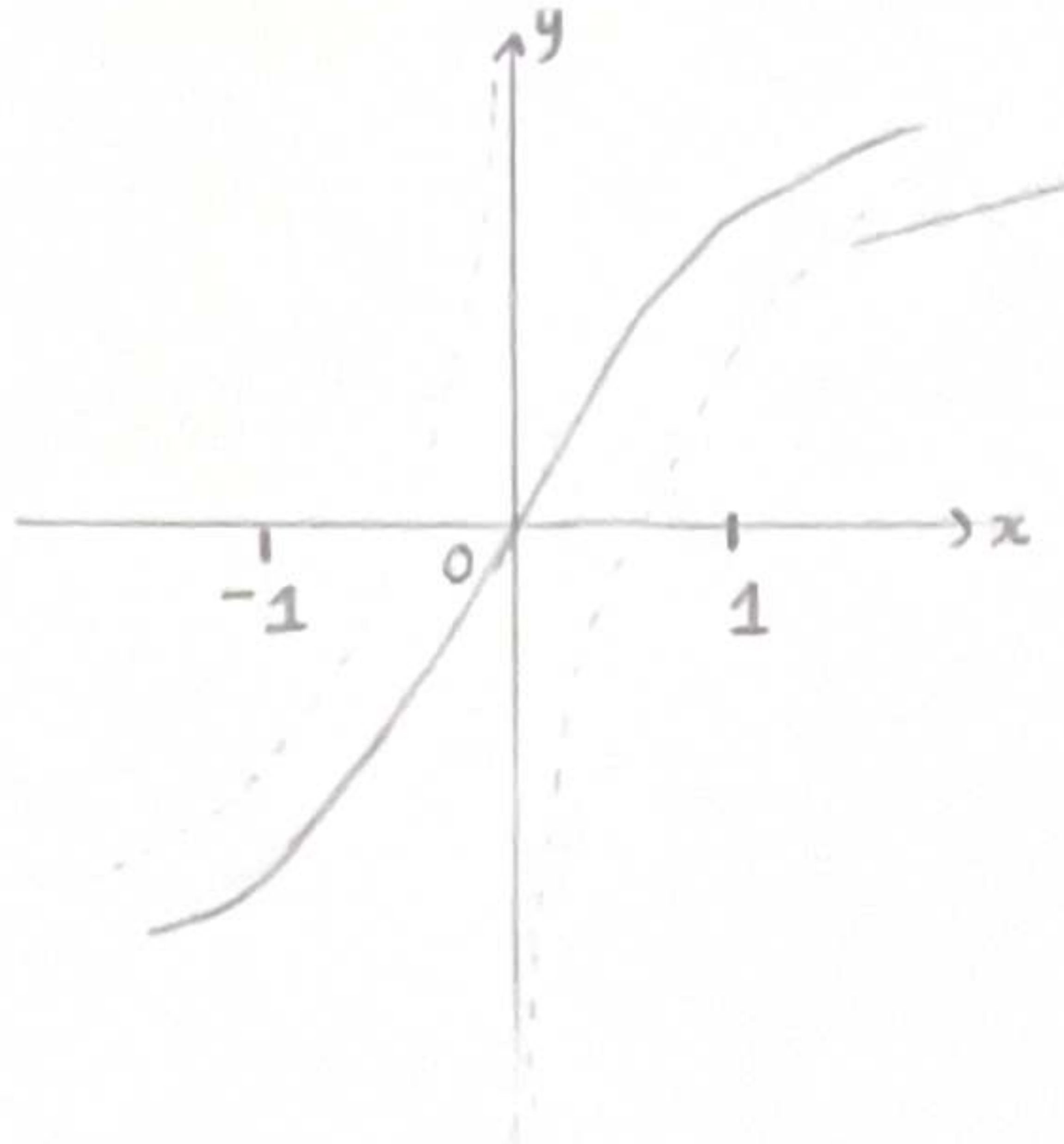


⑮ $y = \tanh(x)$



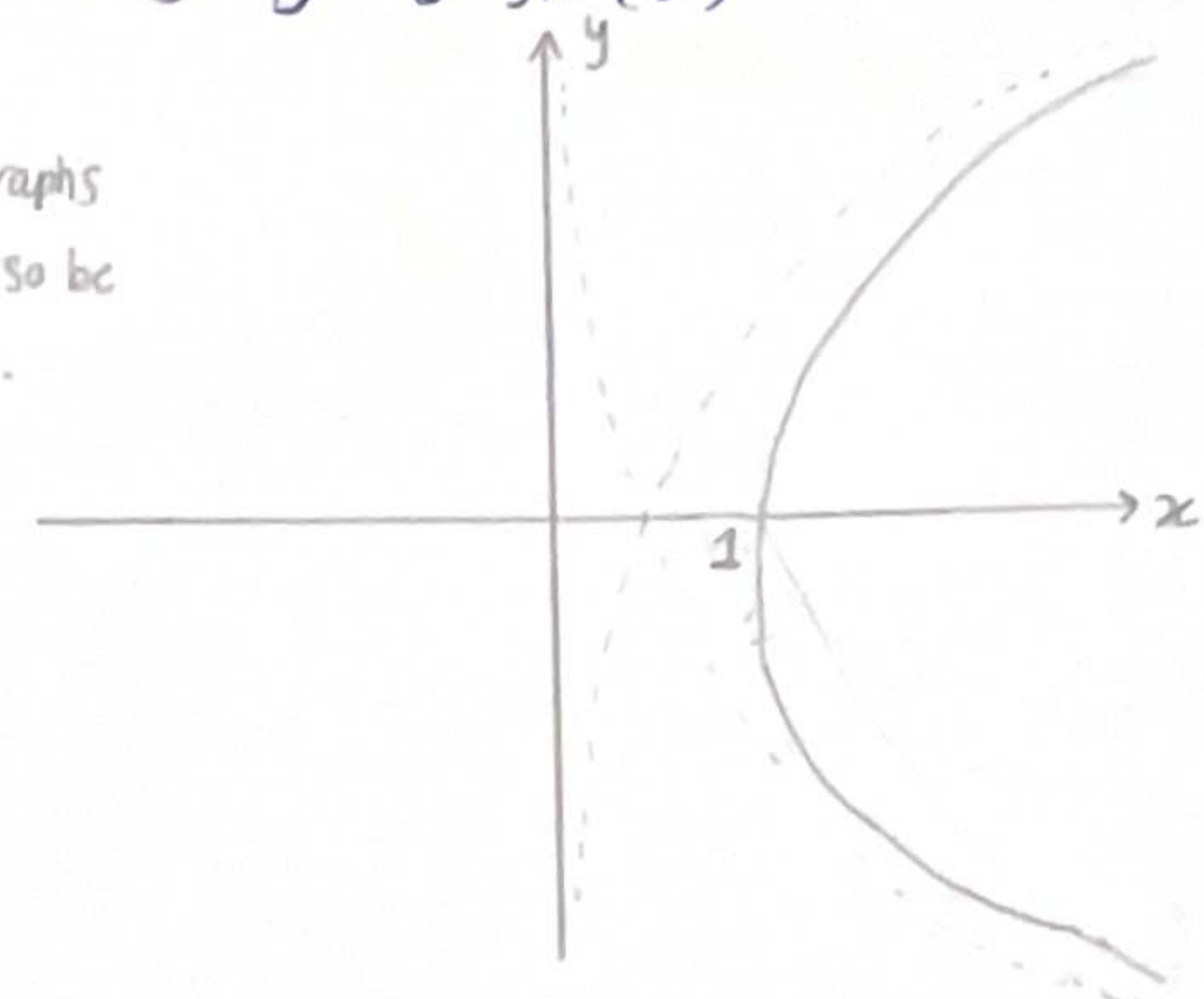
⑨ $y = \cot(x)$

⑯ $y = \sinh^{-1}(x)$

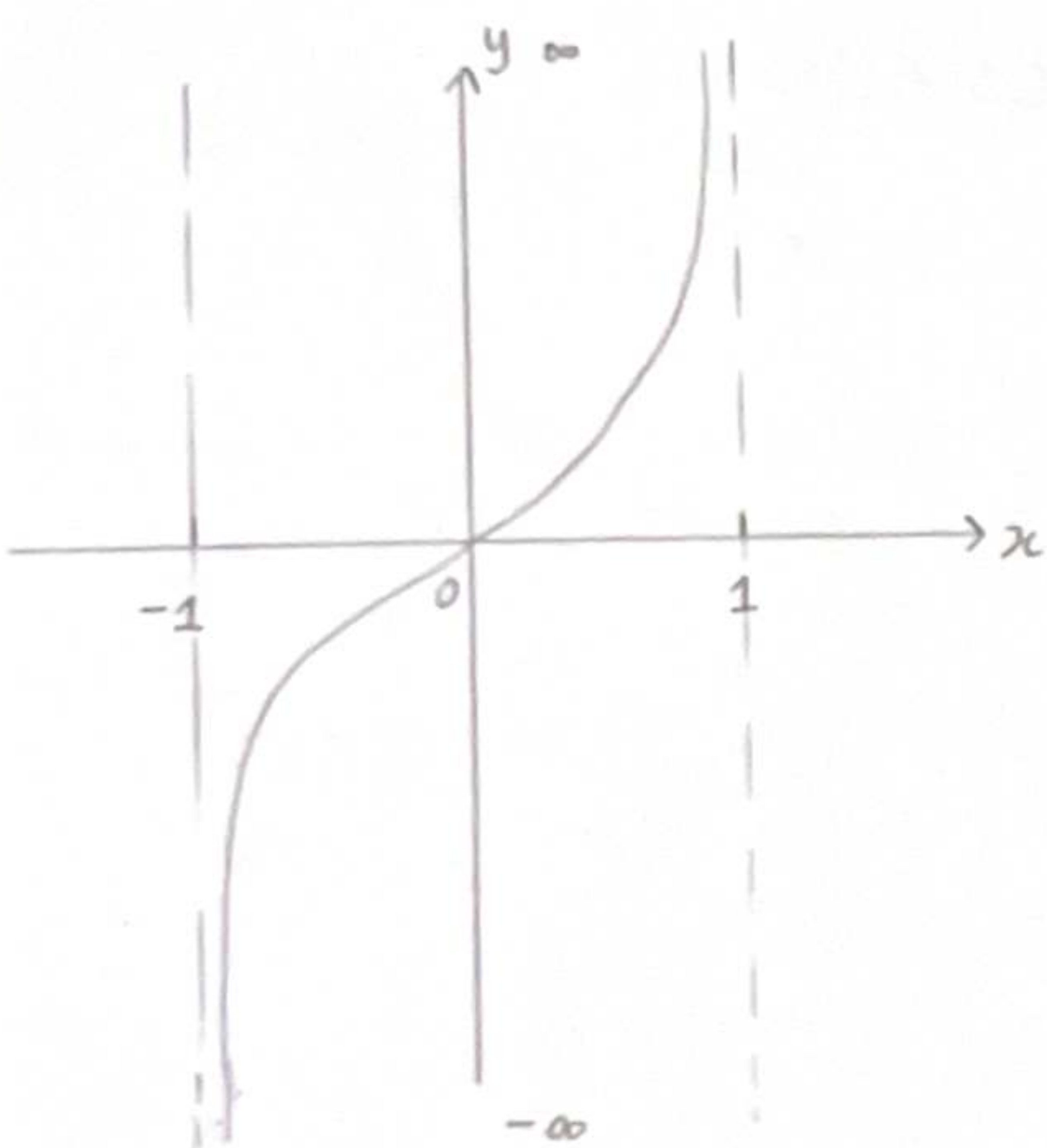


These graphs must also be inversed.

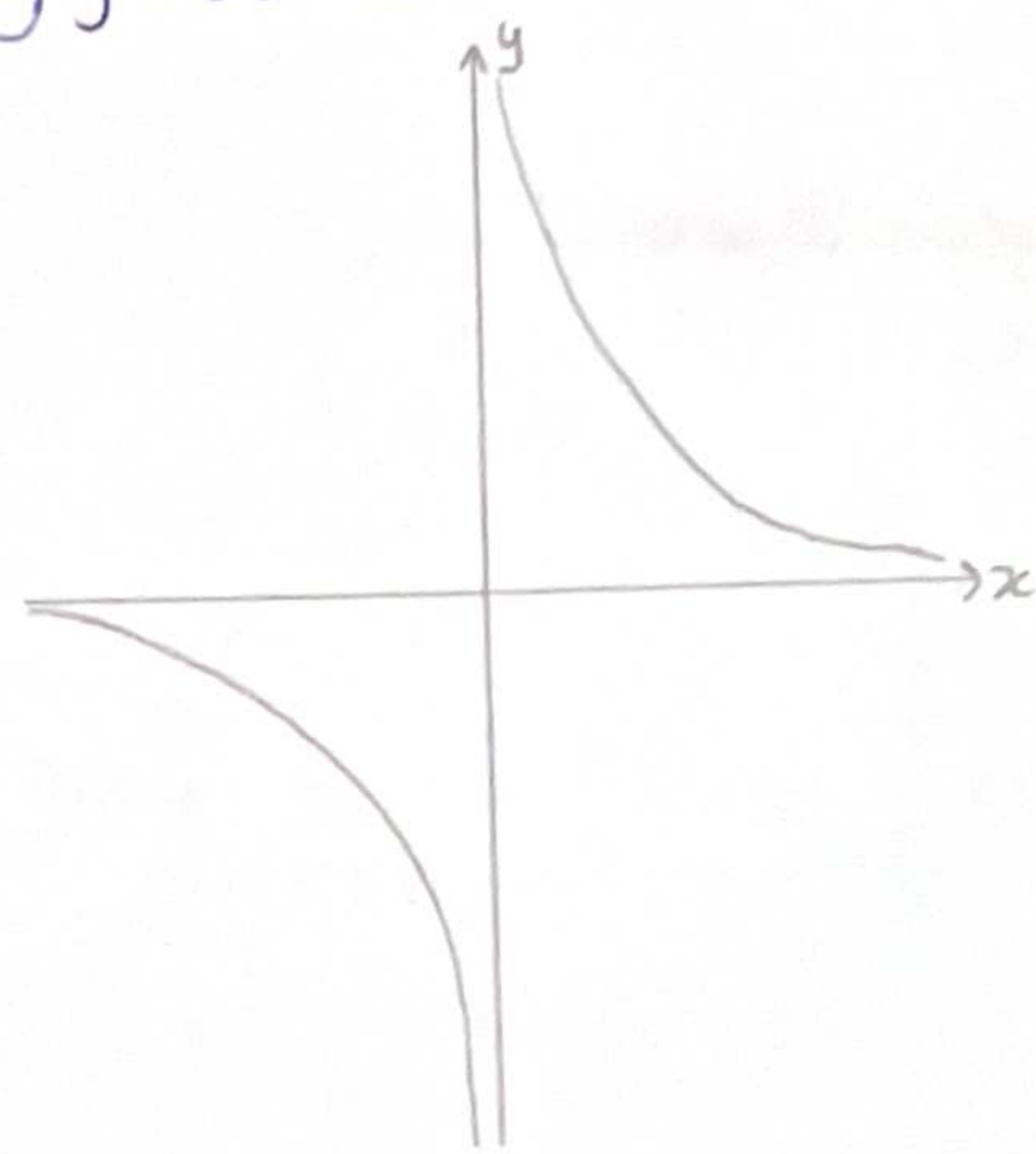
⑰ $y = \cosh^{-1}(x)$



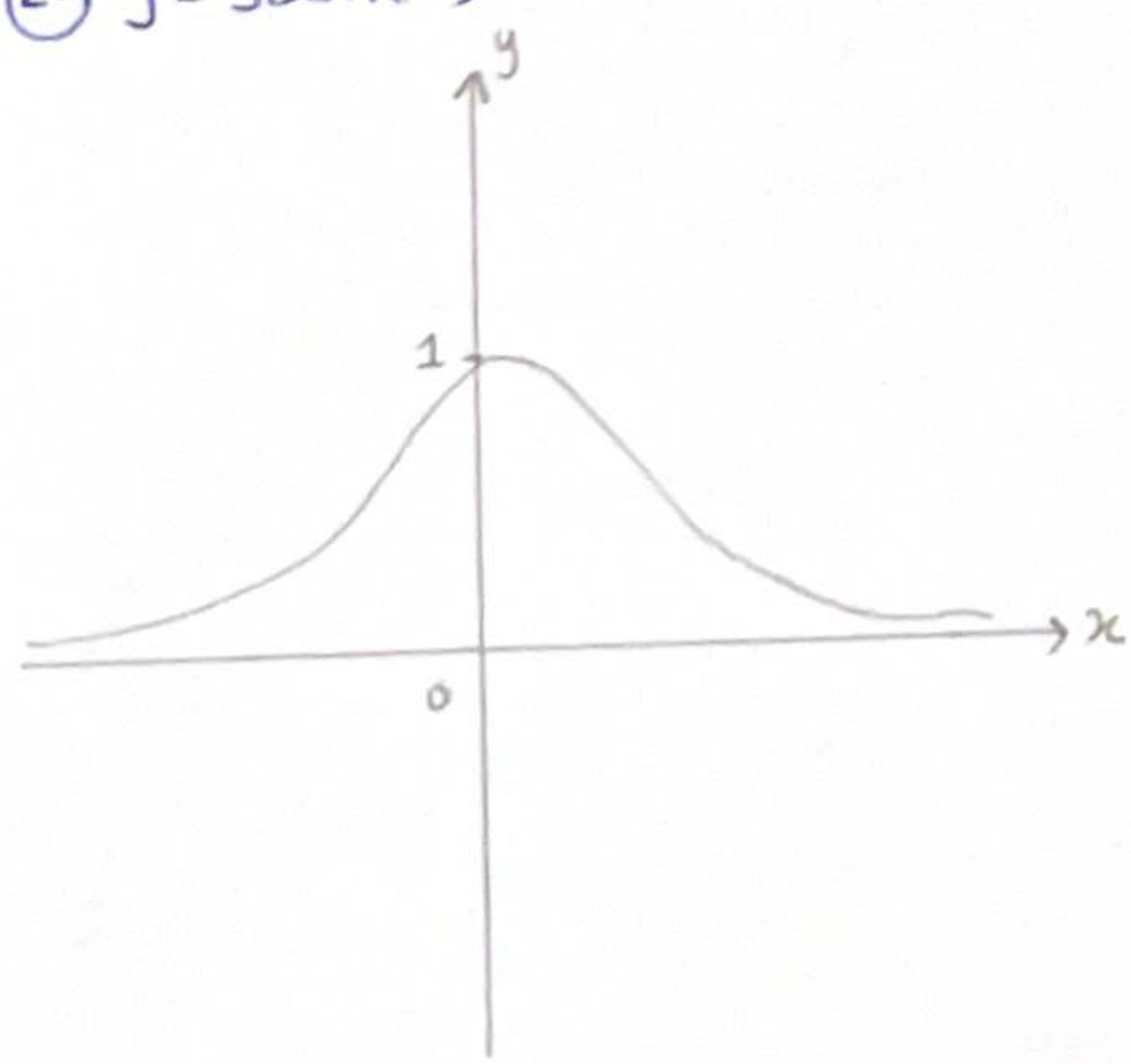
⑱ $y = \tanh^{-1}(x)$



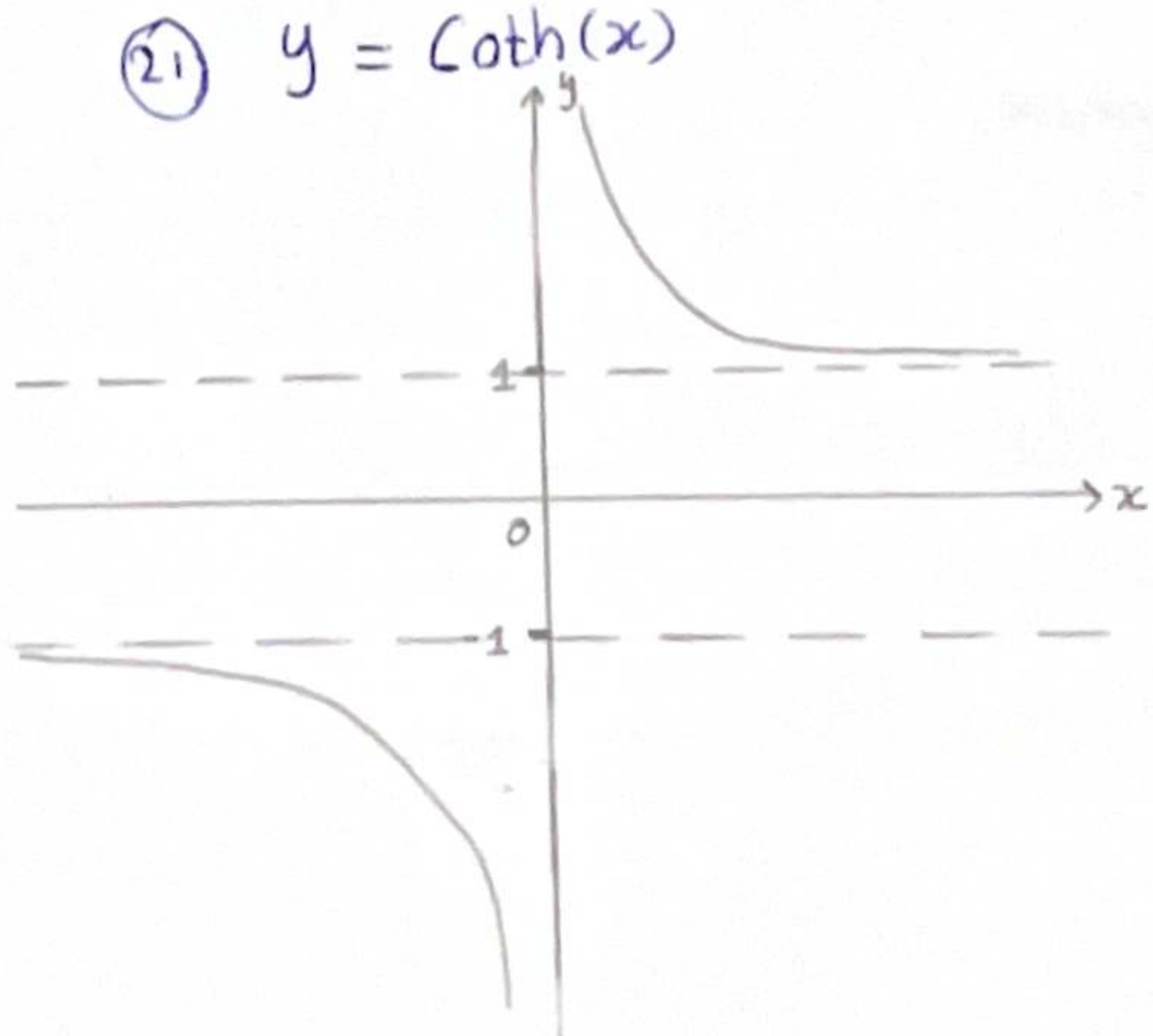
⑲ $y = \operatorname{csch}(x)$



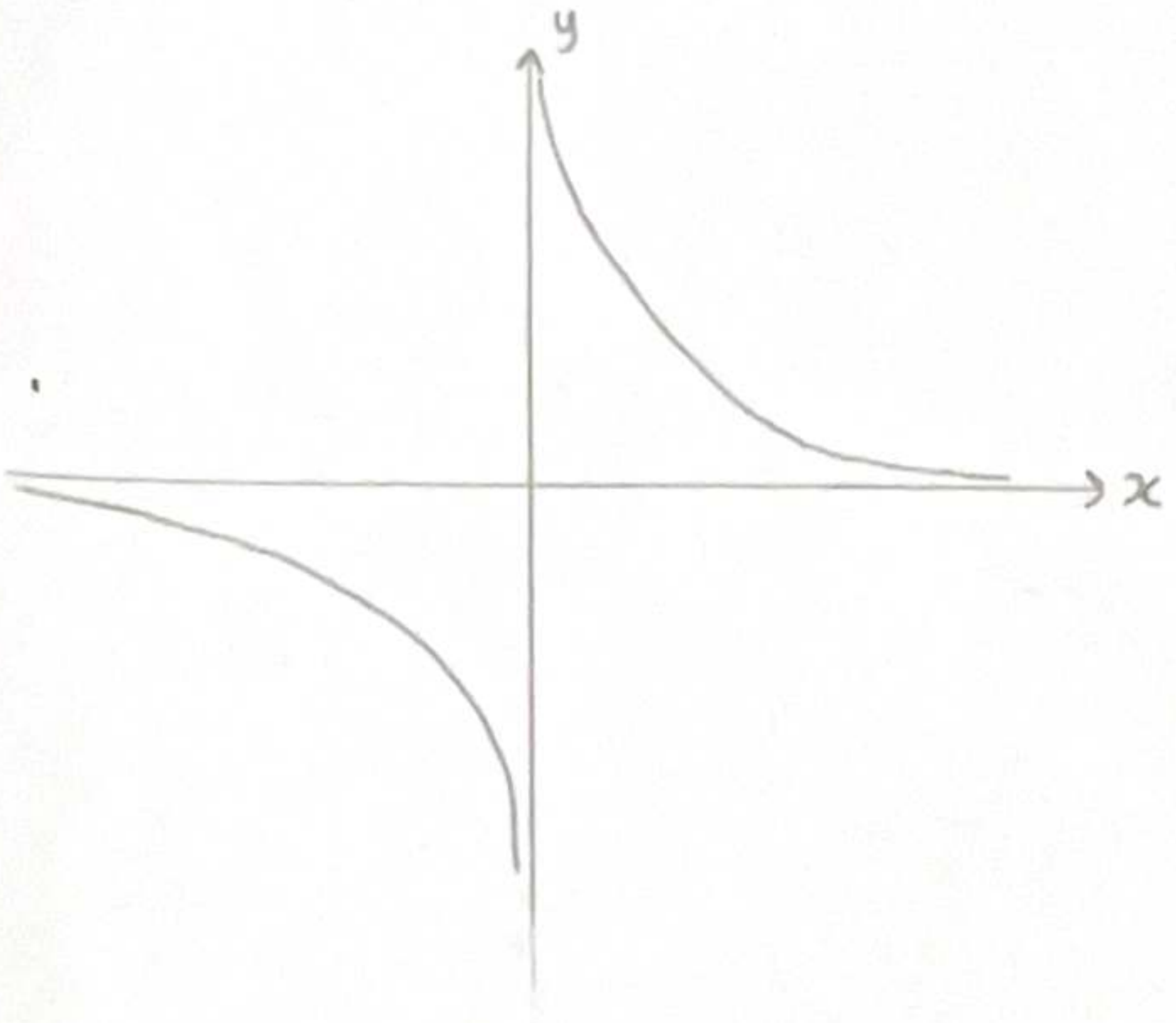
⑳ $y = \operatorname{sech}(x)$



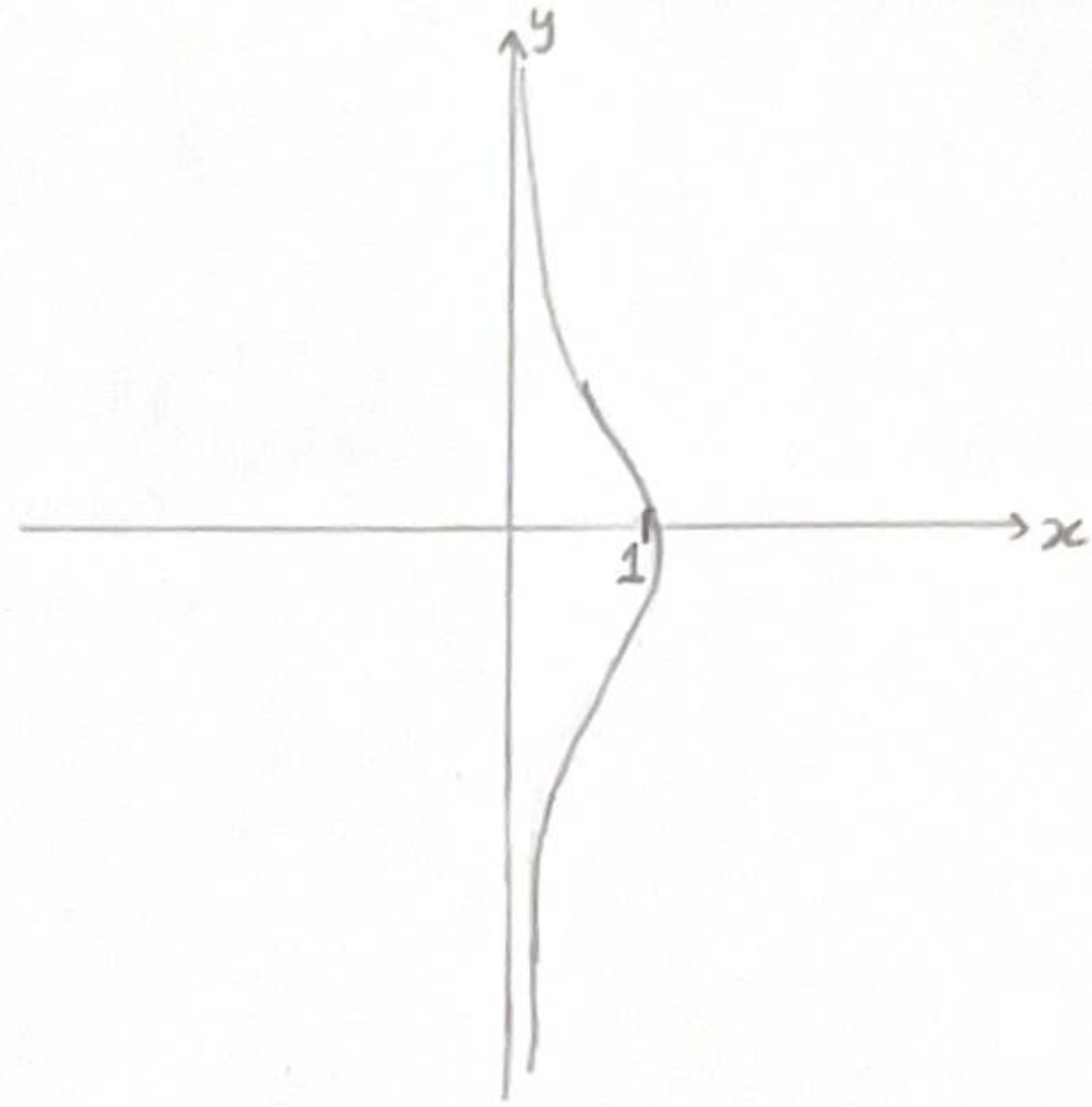
㉑ $y = \coth(x)$



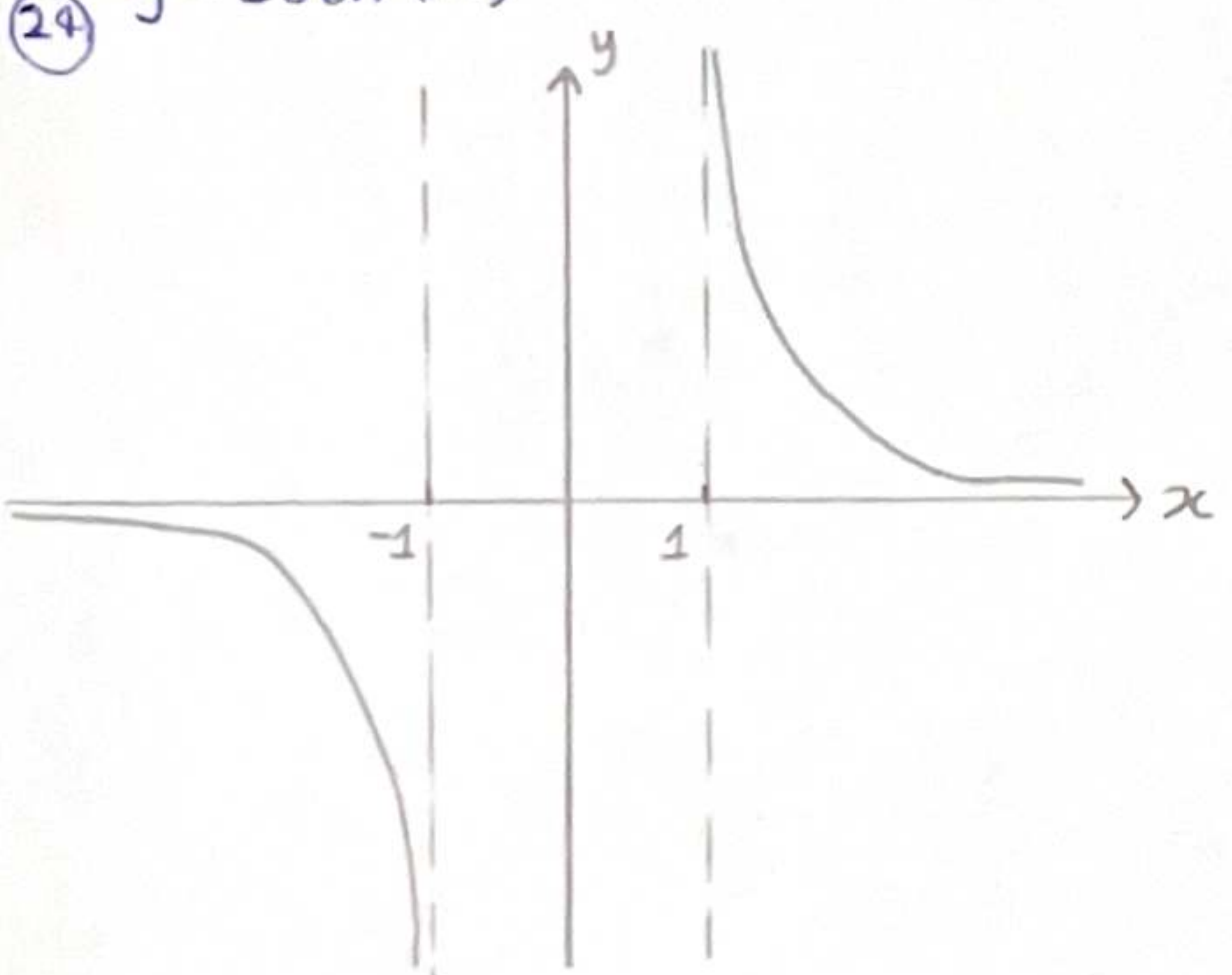
(22) $y = \operatorname{csch}^{-1}(x)$



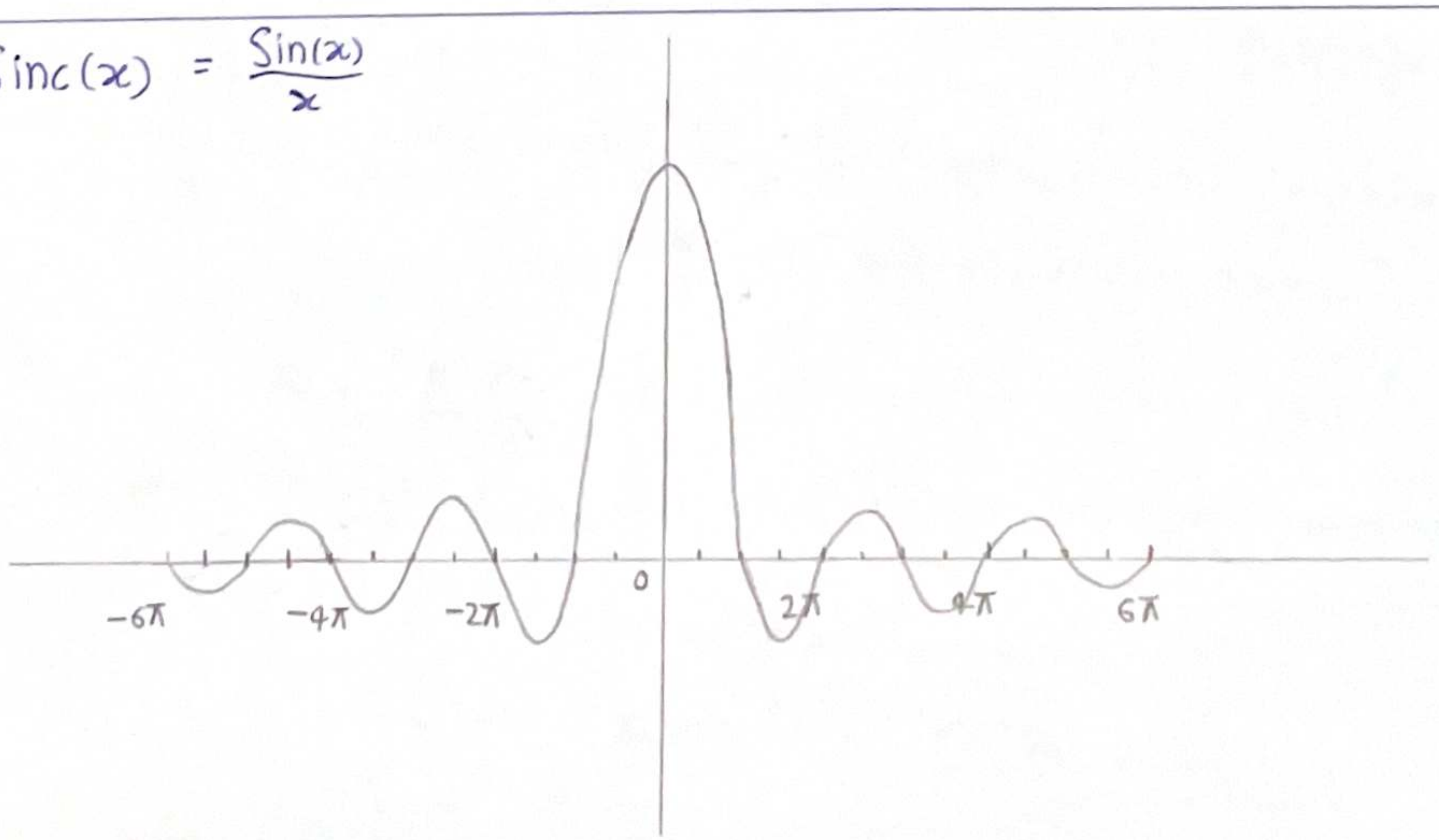
(23) $y = \operatorname{sech}^{-1}(x)$



(24) $y = \coth^{-1}(x)$



$y = \operatorname{Sinc}(x) = \dfrac{\sin(x)}{x}$



4 - Cot(x)

Trigonometric functions on unit Circle.

Latin - Sinus - curve

Co - 'together'

Note Versine

From Latin

'Sinus Versus'

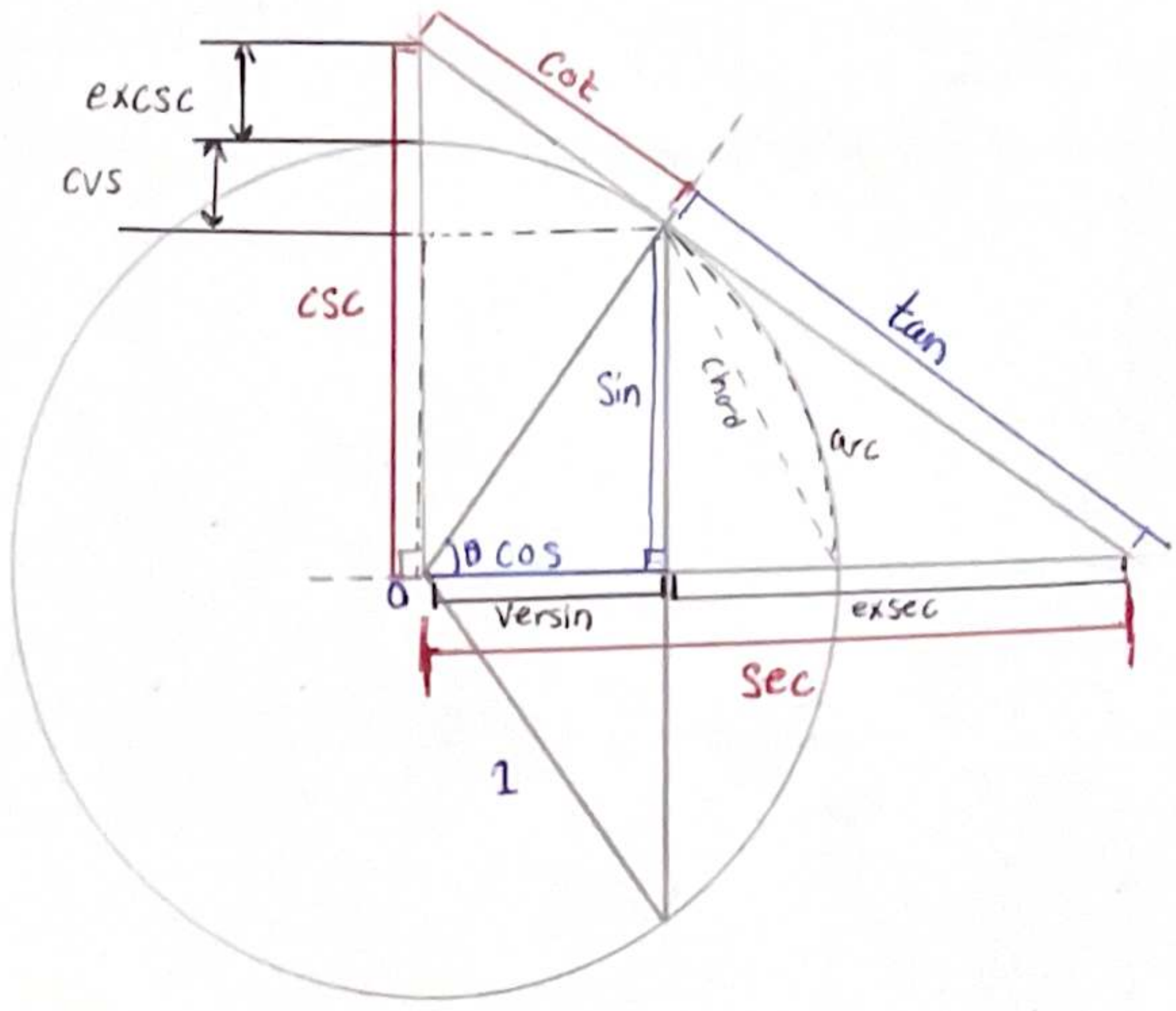
flipped sine.

or vertical Sines

(Sinus rectus)

vertical Cosine

(cosinus rectus)



Other trigonometric expressions.

- Versed Sine, Versine, Versin θ $= 1 - \cos\theta$ $= \text{Ver}\,\theta$

~~Versed Cosine, Vercosine, Coversin θ = 1 - Sin θ~~

- Versed Cosine, Vercosine, Vercosin θ $= 1 + \cos\theta$ $= \text{vcs}\,\theta$
- Coversed Sine, Coversine, Coversin θ $= 1 - \sin\theta$ $= \text{cvs}\,\theta$
- Coversed Cosine, Covercosine, Covercosin θ $= 1 + \sin\theta$ $= \text{cvc}\,\theta$
- half Versed Sine, haversine, have-sine θ $= \dfrac{1 - \cos\theta}{2}$ $= \text{hav}\,\theta, \text{Sem}\,\theta$
- half Versed Cosine, havercosine, havercosin θ $= \dfrac{1 + \cos\theta}{2}$ $= \text{hvc}\,\theta$
- half coversed Sine, hacoversine, cohaver-sine, hacoversin θ $= \dfrac{1 - \sin\theta}{2}$ $= \text{hcv}\,\theta$
- half coversed Cosine, hacovercosine, cohaver-cosine, hacovercosin θ $= \dfrac{1 + \sin\theta}{2}$ $= \text{hcc}\,\theta$
- exterior secant, exsecant, exsec θ $= \sec\theta - 1$ $= \text{exs}\,\theta$
- exterior cosecant, excosecant, excosec θ $= \csc\theta - 1$ $= \text{exc}\,\theta$
- Chord, Crd θ $= 2\sin\left(\frac{\theta}{2}\right)$ ← unit radius (1) or $\text{Crd}(\theta) = 2r\sin\left(\frac{\theta}{2}\right)$