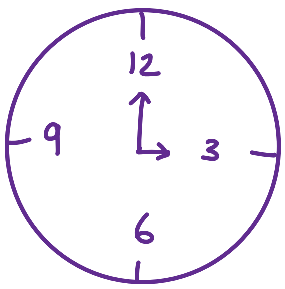
1. Hate Society so much that buy a clock isolated in house & do not obey hour system
2. Count one by one.
3. Wake up one day with numbers inscribed on wall.
4. Decide to contact help
5. Convert e.g. 63 h to actual time.

Person see that activity is repeated

↳ Separate time in equivalence classes.

Hate society so much that even reject clock.



Start at 00 00

Note only the hours.

Or buy no clock ~~in~~ just start counting hours like a mad lad

(Assume I make no errors in keeping track of time)

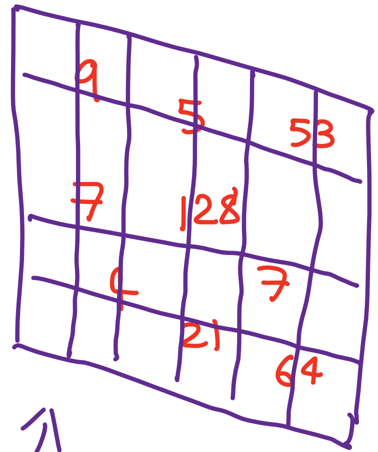
5h	7h	
9h	21h	
4h	53h	on a wall
	64h	
	138 h	

(Include some art)

9 5 7

138 4 21 53

64



Wake up
See these on a wall

Try to derive time

Clock => use mod 24

$4 \bmod 24 = 4$

$9 \bmod 24 = 9$

Explain use

After psychotic / cocaine hangover, wanted to know what happen. Decide to tell some one.

9 → 9
5 → 5
7 → 7
16 → 16
18 → 18

53 → 53 mod 24 = 5

64 → 64 mod 24 → 16

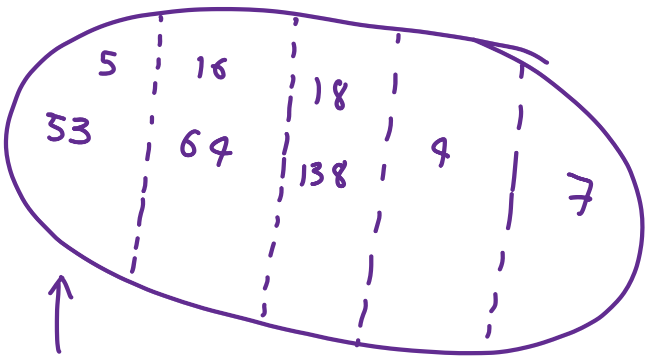
138 → 138 mod 24 → 18

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \end{array}$$

$$\begin{array}{r} 64 \\ -\ 48 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 72 \\ +\ 24 \\ \hline 96 \\ +\ 24 \\ \hline 120 \end{array}$$

Person Helping splits the info on the wall



Equivalence relation on Set "Time on wall"

Notation ?? $5 \sim 53$

[5] [16] [18]

In this case, the Partitions are of different size.

5
$5+24$
$5+2(24)$
⋮
$5+n(24)$

All equivalence classes on the set of integers are of the same size because there is an infinite amount of them.

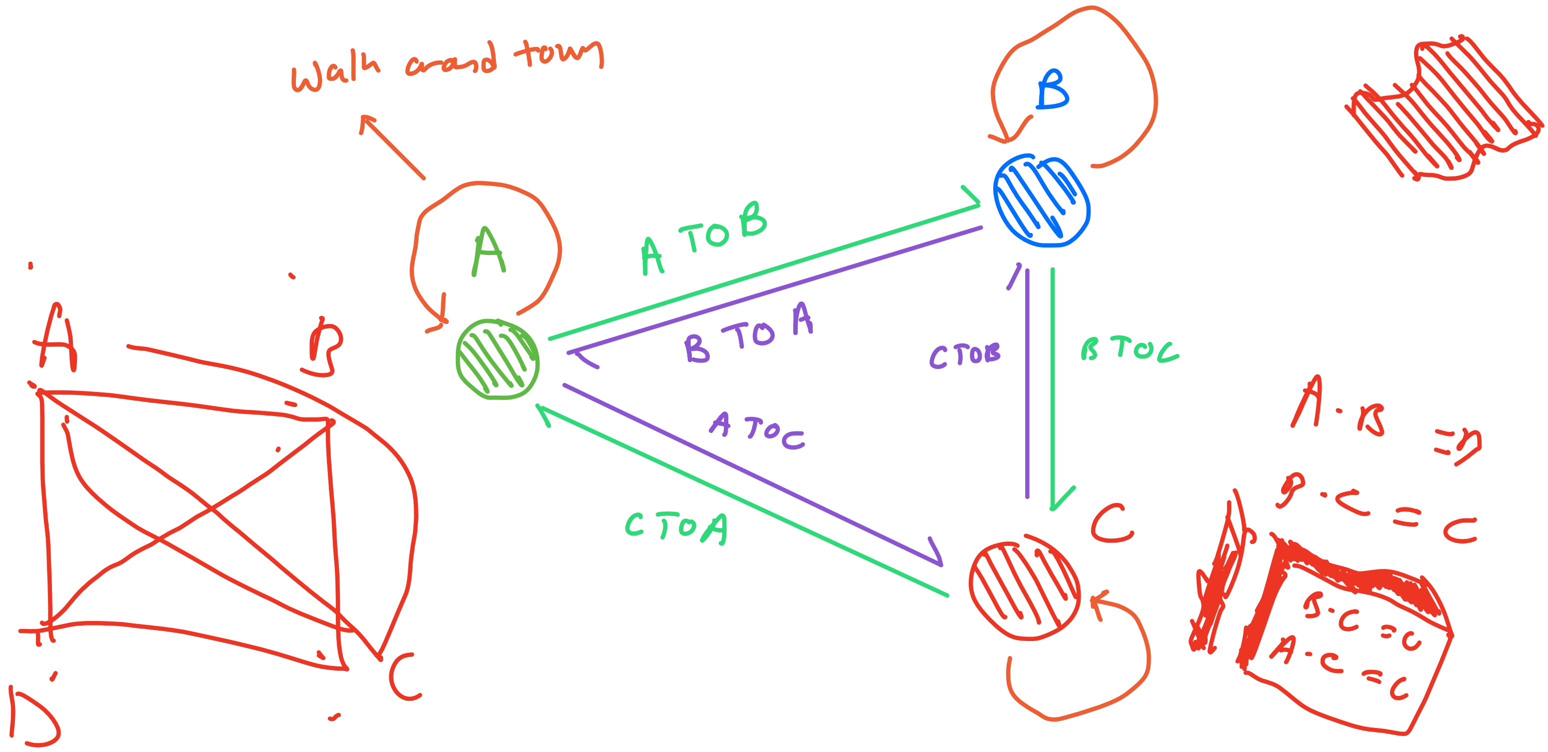
Residue class Integers modulo n.

↳ If an immortal continues my work

e.g. $5 \to 5, 5+24, 5+2(24), \dots 5+n(24)$

We see how equivalence classes Partition the set.

Group



Consider the roads connecting the 3 towns A, B & C. → Indicate how much it reveals the "coolness" of graps.

Let's us look at the roads.

For each town, there is a special "walk around town" road. → This road leaves the destination of the traveler unchanged.

↳ Existence of Identity element.

Stress on how this type of road is same for all towns.

Using the systems of roads provided, we cannot go outside any other town except A, B, C.

↳ Closure.

Associativity

$A \to B \to B \to C$

$A \circ (B \circ C)$

$(A \circ B) \circ C$

Work on Associativity

$A \circ (\underbrace{B \circ C}_{\text{from } C \text{ to } B}) = (A \circ B) \circ C$

$\to A \circ B$

from B to A

$\to A$

From B to A

$A \circ C$

from C to A.

No associativity in road/graph system.

Consider individuals deciding to meet

$(A \circ B) \circ C = A \circ (B \circ C)$

A to B
Bring me to B

Road B to C
Bring me to C.

$\to A \circ C$

Road A to C
bring me to C

Road B to C
bring me to C

form group??

The graphs cannot yield groups.

Isometry on metric spaces

Metric Space : A metric space $M = (A, d)$ is an ordered pair consisting of 1) a non-empty set A together with 2) a real valued function $d : A \times A \longrightarrow \mathbb{R}$ which acts on A and satisfies the Metric space Axioms.

$\hookrightarrow \forall x \in A : d(x,x) = 0$

$\forall_{x,y,z \in A} : d(x,y) + d(y,z) \geq d(x,z)$

$\forall_{x,y \in A} : d(x,y) = d(y,x)$

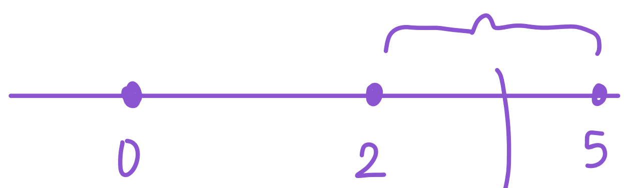
$\forall x, y \in A : \quad x \neq y \Rightarrow d(x,y) > 0$

The elements of A are called the point of the space.

The mapping $d : A \times A \longrightarrow \mathbb{R}$ is referred as a distance fuctin on A or simply distance.

It must not strictly obey the "usual notion" of distance.

$\hookrightarrow$ like consider the set of all points in $\mathbb{R}$



find d

$$A = \{ x \mid x \in \mathbb{R} \}$$

$$d(x,y) = |x - y|$$

$d(2,5) = |2-5| = 3 \longleftarrow$ which is the distance between 2 points.

Metric defines norm in vector space.

↳ State the way we see universe is expanding is because it is the metric which is changing.

Also consider the color of fruits { Oranges, grapes, peach }

d(Orange, peach) < d (Orange , grapes)

↳ where d is some "color closeness" measuring function.

Isometry

Let $M_1 = (A_1, d_1)$ and $M_2 = (A_2, d_2)$ be metric spaces.

Let $\phi : A_1 \rightarrow A_2$ be a <u>bijection</u> s.t :

$$\forall a,b \in A_1 : d_1(a,b) = d_2(\phi(a), \phi(b))$$

↳ The relative "closeness" between the elements are present even after mapping to another metric space.

Decomposable Set

A set $S \subset \mathbb{R}^n$

↑ Real coordinate space. (n^{th} dimensional space)

↑ Part of.

A set $S \subset \mathbb{R}^n$ is decomposable in m sets $A_1, \cdots, A_m \subset \mathbb{R}^n$ if there exist isometries $\phi_1, \cdots \phi_m : \mathbb{R}^n \to \mathbb{R}^n$ s.t

$$S = \bigcup_{k=1}^{m} \phi_k(A_k)$$

$$\forall i \neq j : \phi_i(A_i) \cap \phi_j(A_j) = \emptyset$$

Notes: Both S & $A_1, \ldots A_m$ are in $\mathbb{R}^n$ so we are talking about "factoring" a set with the factors in the same space.

ϕ_1 sends one portion of S to A_1
ϕ_2 sends another portion of S to A_2
$\vdots$
ϕ_n sends another portion of S to A_n

These are the portions mapped by the metric.

$\phi_i(A_i) \cap \phi_j(A_j) = \emptyset \rightarrow$ The portions do not co-incide.

Defn: Equidecomposable

why $\mathbb{R}^n$??

Two sets $S, T \subset \mathbb{R}^n$ are said to be equidecomposable if there exists a set $X = \{A_1, A_2, \ldots, A_m\} \subset \mathcal{P}(\mathbb{R}^n)$ where $\mathcal{P}(\mathbb{R}^n)$ is the power set of $\mathbb{R}^n$, such that both S and T are decomposable in the elements of X

Consider $W = \{1, 2, 3\}$

$\Rightarrow P(W) = \{\{\varnothing\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$
$\{1,2,3\}\}$

Now $X \subset P(W)$ e.g. $\{\{\varnothing\}, \{2\}, \{1,3\}, \{2,3\}, \dots\}$

For equidecomposition of S & T, there must be isometries which are able factor S & T into some sets of the Power set.

i.e. Consider $S \rightarrow$ S must be decomposed (splitted by an isometry) into some subsets of its power set.

e.g. $S, T \subset \{1, 2, 3\}$

e.g. $S = \{1, 2\}$ $T = \{3\}$

Now X is a subset of the Power set $\{1, 2, 3\}$

e.g. $\{\{1,2\}, \{\varnothing\}, \{3\}, \{1,2,3\}\} = X$
$\{1\}, \{2\}$

We must have a function (an isometry) which splits S so as it is represented by the union of some elements belongs in X.

e.g. $S = \{1, 2\}$

In X : $S = \{1\} \cup \{2\}$

or $\{1, 2\} \cup \{\emptyset\}$

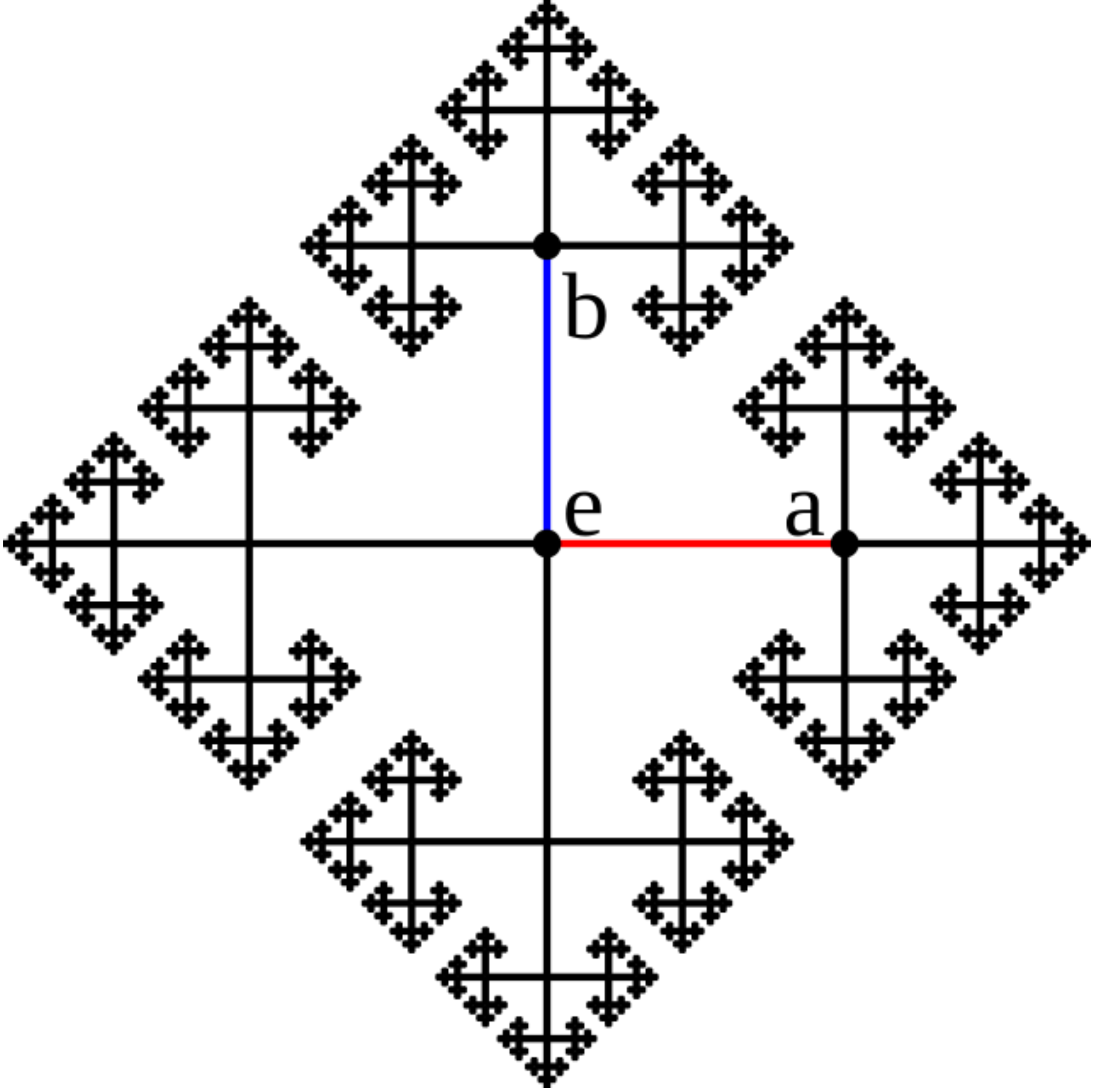
$T = \{3\}$

In X : $T = \{3\} \cup \{\emptyset\}$

$\Rightarrow$ Both S & T are decomposable in the elements of X

BUT They are <u>not</u> equidecomposable as they are not made up of the same elements. $\boxed{??}$

The Free-Group.



F_2 is the set of all words using a, b, a^{-1}, b^{-1} (& e)

Union of all possible words is

$\{\emptyset\} \cup S(a) \cup S(b) \cup aS(a^{-1}) \cup bS(b^{-1})$

All words starting with a

All words not starting with a

$S(a^{-1}) = a^{-1}ba^{-1}aabbabb^{-1}a$

$aS(a^{-1}) = a\cancel{a^{-1}}\,ba^{-1}aab\,bab^{-1}a$

$\cancel{a}\cancel{a^{-1}}\cancel{a}\,a\,bab^{-1}a$

sti

aa still start with a.

The Product of Shapes

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