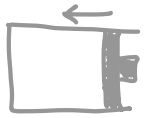
Simple Gas Laws

Boyle's Law : As pressure of gas increases, its volume decreases HOW ??

$$P \propto \frac{1}{V} \equiv PV = K$$

We know how to change the volume of a gas, change container size.



↳ What is pressure and how does it affect volume.

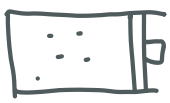
How do we increase pressure ?
↳ ① Decrease volume
② Increase no of molecules
③ Increase temperature in a rigid container.

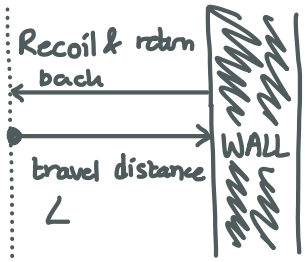
Pressure is a macroscopic phenomena observed when walls hit container wall.

① When we increase no molecules
⇒ more collisions ⇒ more pressure.

② When we increase T w/o changing V
The molecules have more energy but lose none by doing work to change V (move piston).

③ Decrease volume : Collisions are more → frequent





$$\Delta P = P_{ix} - P_{fx} \qquad P_{ix} = -P_{fx} \text{ (Elastic)}$$

$$\therefore \Delta P = 2P_{ix} \text{ (change in momentum)}$$

$$\Delta t = \frac{2L}{V_x}$$

$$F = \frac{\Delta P}{\Delta t} = \frac{2P_{ix} \cdot V_x}{2L} = \frac{mv_x^2}{L}$$

Pressure is Aerial density of molecular force.

Suppose reduce L by ½

$$\Rightarrow \Delta t' = \frac{L}{V_x}$$

$$\Rightarrow F' = \frac{2mv_x^2}{L} \quad (F_{1/2} = 2F_1) \text{ (Pressure doubles)}$$

↳ But how does it affect temperature & molecular velocity?

$V_{m|P} = V_{m|B} + V_{B|P}$

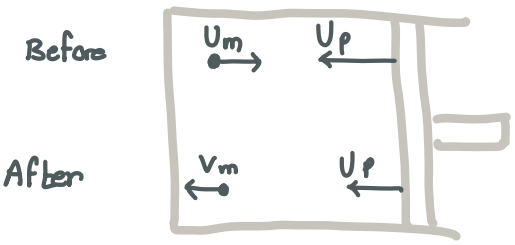
app.

$= U_m + (-U_p)$

$= U_m - U_p$

sep. $-V_m - U_p$

→ (+)



(e)

Elastic Collisions ⇒ Coefficient of Restitution = 1

$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of Approach}} = 1$$

$$\frac{V_m - U_p}{U_m + U_p} = 1$$

$$\Rightarrow V_m - U_p = U_m + U_p$$

$$\Rightarrow V_m = U_m + 2U_p \quad / 2U_m + U_p.$$

??

Clarification on Vel. app & Vel. sep.

Relative Velocity $\vec{V}_{B|A}$ (Vel. of object/observer B in rest frame of another object/observer A)

→ (+)

Approach $\vec{V}_{M|P} = \vec{V}_{M|B} + \vec{V}_{B|P}$

$= U_m + U_p$ ↘ $= -V_{P|B}$

(Define +ve sense)

$c \gg V_m, U_m, U_p$

Separation : $V_{m|B} - V_{P|B}$

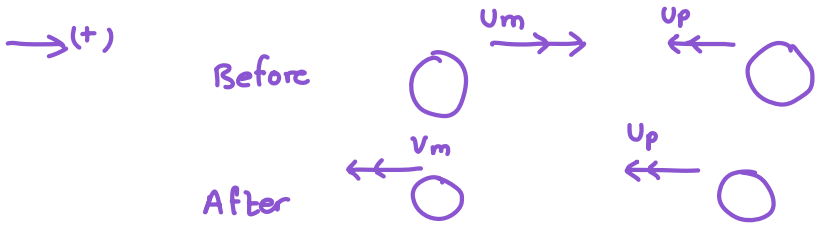
$= -V_m - U_p$

$= -(V_m + U_p)$

$\vec{V}_{M|P} = \vec{V}_{M|B} + \vec{V}_{B|P} = -V_{P|B}$

- $\vec{V}_{M|P}$: Vel. molecule relative to piston.
- $\vec{V}_{M|B}$: Vel. molecule relative to box
- $\vec{V}_{B|P}$: Vel. Box relative to piston
- $-V_{P|B}$: Vel. Piston rel. to box.

→ (+)

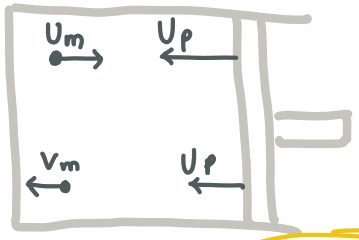


$$m_m U_m - m_p U_p = -m_m V_m - m_p U_p$$

$$m_m (U_m + V_m) = m_p (U_p - U_p)\ U_m - V_m$$

(perfectly Elastic Collisions)

P is in rest frame of M



(+)

$\vec{V}_{m|P} = \vec{V}_{m|B} + \vec{V}_{B|P}$

$V_{P|m} = V_{P|B} + V_{B|m}$

$= +U_p + U_m$

Sep $= +U_p - V_m$

$V_{B|m} = -V_{m|B}$

$V_{m|P} = V_{P|B} + V_{B|m}$

$= V_{P|B} - V_{m|B}$

App $= -U_p - (+U_m)$

$= -U_p - U_m$

Sep. $= -U_p - V_m$

$V_m - U_p = U_m + U_p$

$V_m = U_m + 2U_p$

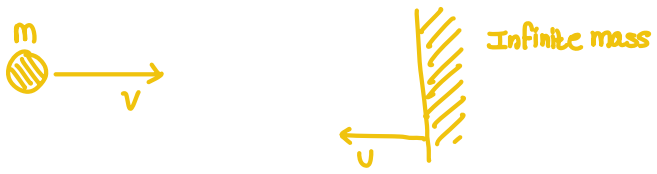
A ball of mass m moves towards a moving wall of infinite mass with a speed V along the normal to the wall. The speed of the wall is U towards the ball. The speed of the ball after elastic collision with the wall is:

A) U+V away from the wall

B) 2U+V away from the wall ✓

c) |U-V| away from the wall

D) |V-2U| away from the wall

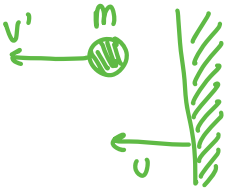


Soln:

Elastic (e=1)

Velocity before & after coll. is same

Consider FOR of wall



Total Vel before collision = U+V

After collision = U−V'

$\Rightarrow U+V = -(U-V')$

↑ ball change direction

$U+V = -U+V'$

$\Rightarrow \boxed{V' = 2U+V}$ (B)

Relative Velocities

$V_1 - V_2 \Rightarrow U-(-V) = U+V$ (Before)

$\Rightarrow U-V'$

↳ change direction

$-(U-V')$

$= V'-U$

Use Notation: $V_{A|C} = V_{A|B} + V_{B|C}$

See how to make it apply.

(1) (1) (1) (1) (1)

+0.2 +0.2 +0.2 +0.2 +0.2

1.2 1.2 1.2 1.2 1.2

No change in dispersion of molecular Energy.

(2) (1) (3) (1) (4) = 11

+0.2 +0.2+0.2+0.2 +0.2 +1

No change in initial dispersion of molecular Energy ——— 12

$\Rightarrow \Delta S = 0$ (W)

Dispersion change can only take place by a heat transfer.

Energy grading.

Work

Doing (macroscopic) work transfers energy in a fashion such that all the (x-component) molecular energy are raised by mv_p^2 for all molecules.

To reject the energy added in form of work, the opposite occurs. The molecules get rid of their velocities but directs it opposite to the initial one, effectively exerting a macroscopic force in the opposite direction.

A gas piston behaves as a spring. → (Wave Equation derivation
↳ Air particles Hooke's law.)

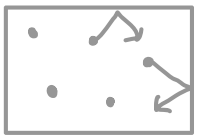
Heat

Heat transfer comes with an increase in Entropy.

Physical meaning of Entropy

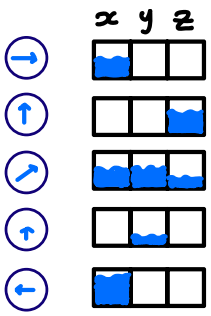
Entropy first appeared as a concept in the field of Thermodynamics. It encompasses a detailed study of energy transfer in the forms of **Heat & Work**. Entropy change always accompanies heat exchange. Let's have a closer look :

For Simplicity, Consider this box filled with 5 ideal gas molecules. They move in random directions with no intermolecular forces.

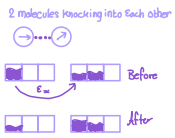


- The internal energy U of the system depends solely on the Temperature which itself is a measure of the average velocity of all molecules. $U = \frac{3}{2} N KT$ (for N molecules).

- The molecules can move freely in the x, y, z direction. Each represent a degree of freedom with an associated translational kinetic energy. (When system is in thermodynamic equilibrium, molecules have $\frac{1}{2} KT$ per d.o.f – Equipartition Theorem).



- Molecules moving only in x will have energy only in that box.
- Diagonal velocity represented by all boxes filled.
- Lower speed is less energy in box.



Note : The molecules constantly bump into each other, changing the state of the boxes. (empty, half, quarter-filled,..) The same occurs at thermodynamic equilibrium at temp. T. Each bump exchanges $\frac{KT}{2}$ energy in x, y or z. On large scale, we would see molecules having $\frac{KT}{2}$ in all boxes. (Average over time).

Now, the kinetic energy possessed by a molecule is the sum of E_K in all directions, i.e. $\frac{3}{2} KT$. For our 5 molecules, let it be as such :

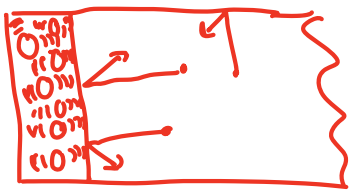
1J 1J 1J 0.2J 1.8J

2 Methods of Energy Transfer, $\Delta U = \Delta Q - \Delta W$ (First Law of Thermodynamics)

HEAT

- We will wrap our box with a heated blanket at temp. T.
- This will cause the molecules in the wall to vibrate at temp. T too.
- For simplicity, we shall transfer 1J (Q = 1J) and consider only one side of the box.

Heating the box walls causes the molecules in the walls to vibrate more violently. This is due to their increased kinetic Energy. The vibrations occur in a RANDOM manner.



There are 2 things : The gas molecules arrive in random directions.

(0) There is more Energy
⇒ More ways to distribute it.
↓
Work?

(i) Due to randomness of collision, a gas molecule arriving in the x-direction might get some Energy in the y direction. (jiggling)

(ii) ?? ↓ Yes, M.B distribution at max Entropy. ↑ N.D

As for the amount of Energy received, it is also random, 2 molecules in same state might get different Energies, one 0.2 J, another 0.001 J

(i) The distribution of Energy changes.

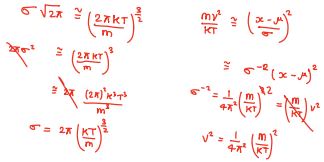
look at what r.m.s does.

Maxwell-Boltzmann Distribution p.d.f (3-dimensional Velocity Distribution)

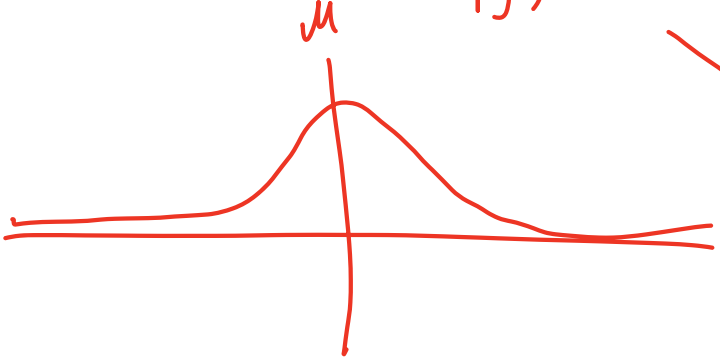
$$f(\vec{v}) = \frac{1}{(2\pi kT/m)^{3/2}} e^{-\frac{m\vec{v}^2}{2kT}}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f(x) = e^{-x^2}$$



We notice that at Maximum Entropy, the Velocity distribution becomes normal.



→ We know heat transfer increases Entropy.

⇒ Heat affects the distribution of velocity.

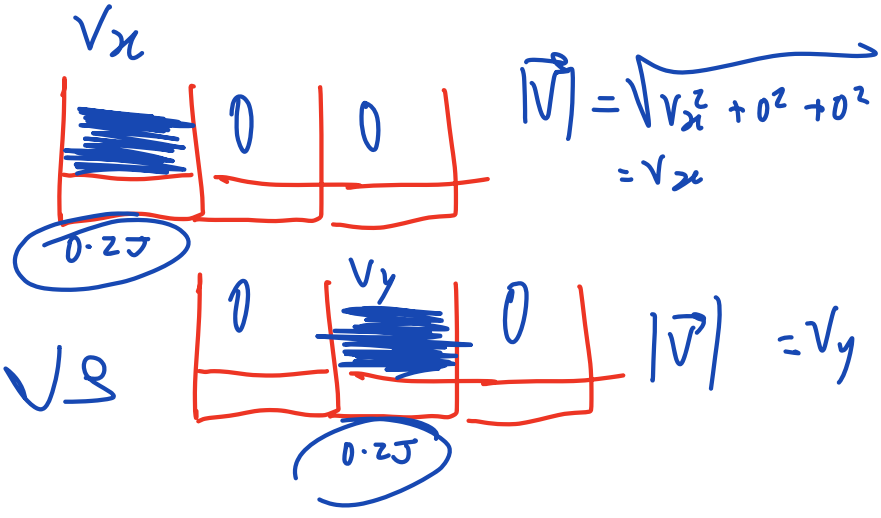
Like Galton-Board, The randomness cause Normal distribution.

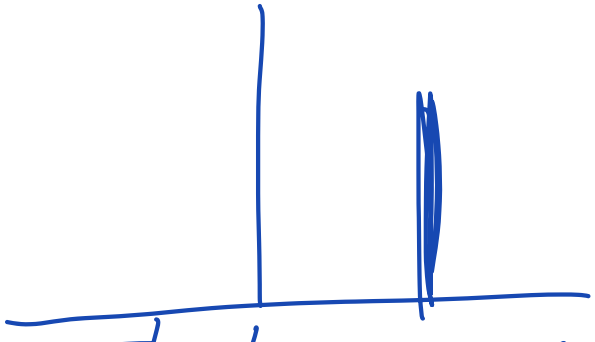
$f(\vec{v})$ is for 3D velocity

$\vec{v} = v\hat{i} + v\hat{j} + v\hat{k}$ $\quad |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$E_k = \frac{1}{2}m|v|^2$

⇒ The Whole $\frac{3}{2}kT$ itself is being randomly distributed.





The distribution would look the same after heating. Only the particles

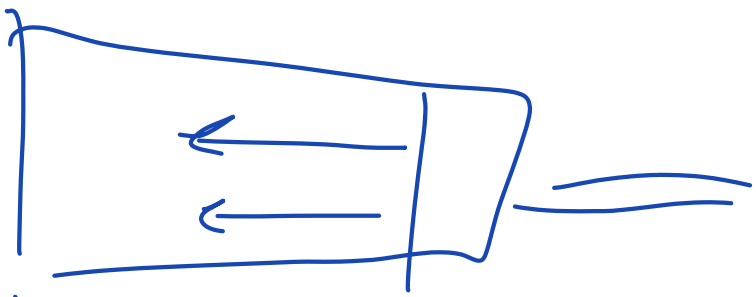
would move in different directions.

Heating would not give bell curved shape.

⇒ Both $\frac{3}{2}$ KT is distributed (random Energies)

& random direction assigned.

Work →



No change in Entropy

⇒ Distribution remains the same.

All Particles get same Energy & also in same d.o.f.

What if get same Energy in different degrees of freedom?

or different Energies in same degree of freedom?