~~Ends~~

Ohm's law : $\vec{J} = \sigma(\vec{E} + (\vec{v} \times \vec{B}))$

Ampère Maxwell law : $\nabla \times \vec{B} = \mu_0 \vec{J} + \cancel{\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$

Displacement Current is 0 negligible in Plasma

Plug Ohm's law in Ampère-Maxwell

↳ $\nabla \times \vec{B} = \mu_0 \sigma (\vec{E} + (\vec{v} \times \vec{B}))$

$$\vec{E} + (\vec{v} \times \vec{B}) = \frac{1}{\mu_0 \sigma} \nabla \times \vec{B}$$

Faraday Law : $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{E} = \frac{1}{\mu_0 \sigma}(\nabla \times \vec{B}) - (\vec{v} \times \vec{B})$$

$$\nabla \times \vec{E} = \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B}) - \nabla \times (\vec{v} \times \vec{B})$$

↳ To get rid of that thing, use Levi-Civita identity & its relation to delta kronecker

$$\begin{aligned}
\nabla \times (\nabla \times \vec{A}) &= \varepsilon_{ijk} \varepsilon_{kmn} \partial_m A_n \\
&= \varepsilon_{kij} \varepsilon_{kmn} \partial_j \partial_m A_n \\
&= \partial_j \partial_i A_j - \partial_j \partial_j A_i \\
&= \partial_i \partial_j \partial_j - \partial_j \partial_j A_i \\
&= \nabla(\nabla \cdot A) - \nabla^2 A
\end{aligned}$$

$$\Rightarrow -\frac{\partial \vec{B}}{\partial t} = -\frac{1}{\mu_0 \sigma} \nabla^2 B + - \nabla(\vec{v} \times \vec{B})$$

$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}$$

$$\Rightarrow \nabla \times \vec{E} = \frac{1}{\mu_0 \sigma}\left(\nabla(\underbrace{\nabla \cdot B}_{\to 0}) - \nabla^2 B\right)$$

No Magnetic Monopoles Gauss's Law

Frozen in Flux Theorem

$$\left[\frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{v} \times \vec{B})} + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}}\right.$$ → Induction equation governing magnetic field diffusion in solid / liquid (gas) → plasma. fluid

Induction Portion
$\underbrace{E = \vec{v} \times \vec{B}}_{\text{Lorentz force}}$
Curl of E
~~is mag~~ like Maxwell Equations.

Diffusion Portion. → lost to heat.

Assume conductivity is uniform throughout fluid / Solid.

How to get this big Guy?

Ohm's Law : $\vec{J} = \sigma(\vec{E} + (\vec{v} \times \vec{B}))$

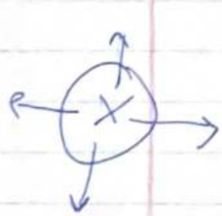
Plug in Maxwell Equations ~~III~~ ~~Vs~~ IV → Ampère Maxwell

Then Plug that in Maxwell Equation III → Faraday's law

↳ Get rid of Cross products using Levi-civita symbol and convert to dot (delta kronecker)

$E = v \times B$

→ ~~Gauss's~~ No magnetic Monopole.
Using $\nabla \cdot \vec{B} = 0$ (Maxwell II) & Vector Calculus, you get the equation above.



Dropping Some terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times B) + \cancel{\frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}}$$

Dropped if $R_m \gg 1$

When Conductivity tends to ∞, diffusive terms drop.
Molten Core, Sun, Plasma

Effect → Magnetic fields cannot diffuse out & remain frozen in the fluid.

When Conduction $R_m \ll 1$, Diffusion of magnetic field is like heat equation.

$\nabla^2 f = 0$

Real valued

$$\boxed{\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial t^2}} \rightarrow \sim \text{Laplace Equation}$$

↓ vector valued + in 3D

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}} \rightarrow \text{Like Laplace Equation.}$$