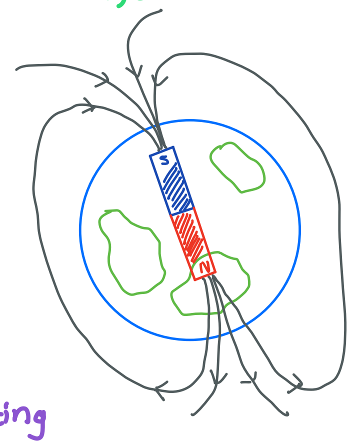
The magnetic field of The Earth.

- The Earth has a magnetic field which extends from its core out to Space. The magnetic field near the surface can be viewed as a very big bar magnet through Earth, tilted 11° w.r.t Earth's rotational axis. [Dipolar Approximation].



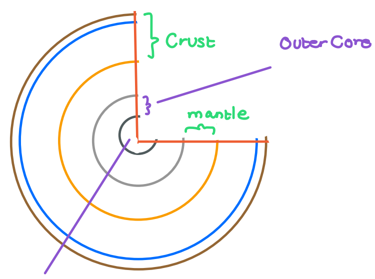
Dynamo Theory

It provides a model to try and Explain the Origin of Earth's magnetic field.

The dynamo theory describes a process through which a rotating, convecting and electrically conducting fluid medium can maintain a magnetic field over astronomical time scales. [Also for Jovian Planets & Mercury]

The Outer Core of the Earth.

- Temperature about 3000 - 4500 K in outer regions and 4000 - 8000 K near inner-core side. Mag. field there is 2.5 mT (X 50 stronger than at Surface)
- The outer core is not under enough pressure to solidify; so even if its composition is mainly iron & nickel like the inner-core, it remains mostly liquid.

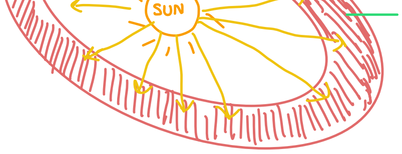


Why does the Core remain hot?

1) Heat when planet formed and accreted (heat not yet lost)
2) frictional heating caused by denser core material sinking towards centre.
3) Heat from decay of radioactive Elements.

1) Planetary Formation (and Heat)

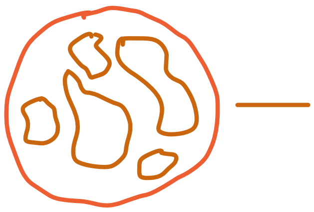
- Before the formation of the Planets, our solar System was like that: a hot gas cloud orbiting



Accretion Disk (Big Cloud)

our planet.

- In that disk, some of these hot particles decided to collide resulting in a big molten rock made up of a mass hot materials.



Young Molten Earth still in accration disk.

- Since the planet is Volumetric, some of the particles "inside" are shielded from the heat of the sun, leaving only the surface of the newly formed planet to recieve the heat of the sun.

As other planets form, accration disk dissociates and heat is less evenly distributed

- The Earth is no longer in thermodynamic equilibrium with the now gone, accretion disk. It thus now starts to cool down by radiation. (to empty space)

Cooldown-Throttling. & Uneven Cooling. → Heat equation.

When Earth was still in its young-molten state, it was made up of "rocks" made of different elements evenly distributed throughout the sphere. As time passed the heavier elements sank to the centre due to density differences. This compositional convection brings about friction which generates additional heat in the planet. ↘ 2000 K contribution

10 000 K contribution.

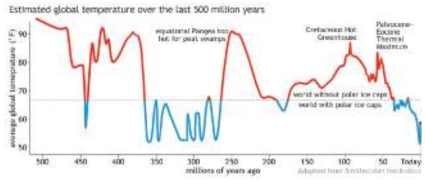
Also, the heat escapes from the centre very slowly as the Earth is massive and the flow of the convectional currents are slowed as they have to travel through a large amount of matter before escaping to the surface. The primordial heat remains trapped in the inner layers.

Furthermore, the insides of the Earth has radioactive elements which release heat energy upon decay. (Potassium, Uranium, Thorium decay) – Accounts for a smaller amount.

↳ Also Asteroid collision which generated

Law of Cooling

$$\frac{dQ}{dt} = kA(T - T_R)$$



the moon/killed dinosaurs provided additional heat to Earth. (Friction)

Like in En loss momentum exercises. (inelastic)

$$\Delta Q = mc\Delta T \Rightarrow dQ = mc\,dT$$

$$\frac{mc\,dT}{dt} = kA(T - T_R)$$

where m : $M_\oplus$

C : Specific heat capacity Earth

A : $SA_\oplus$

k : heat transfer coefficient.

must get better data!

Present Geothermal temperature of Earth = 16°C

At Start molten Earth was about 1982°C.

For all layers in each period & Average them.

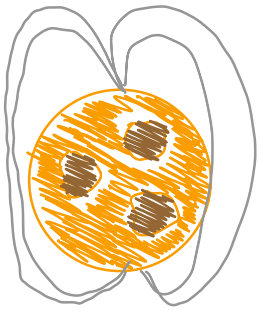
WRITE CONCLUSION.

Back to Magnetism.

We know that ferromagnetic materials in the core cannot account for the geomagnetic field as core temperature is well above the Curie point of Iron (570°C). Rocks & minerals retain magnetism below the Curie point.

The dynamo Theory provides a mechanism for the generation & maintainance of a geomagnetic field.

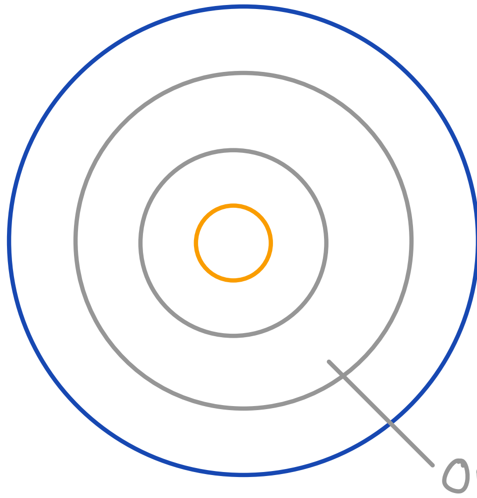
1. An electrically conductive fluid medium.
2. Kinetic Energy provided by planetary motion
3. An internal source of energy to drive convective motions within the fluid.
4. A seed magnetic field.



SUN

Generates its own magnetic field

(acts as seed magnetic field for Earth)



EARTH.

Outer Core

Containing molten iron (Kept molten by high temperature inside). – See maintainance of geothermal heat.

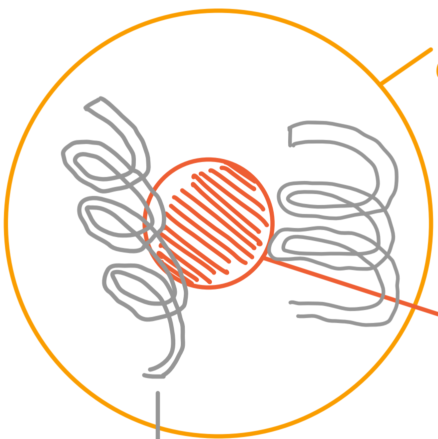
→ Inner core deeper

=> More pressure & more "inside"

=> less heat escape

=> hotter.

Mechanism



Outer Core

Inner Core (Provides heat for convection of liquid iron in outer core)

→ Due to Coriolis Effect (Rotation of the Earth)

the flow of liquid metal is organised into rolls (Taylor Columns)

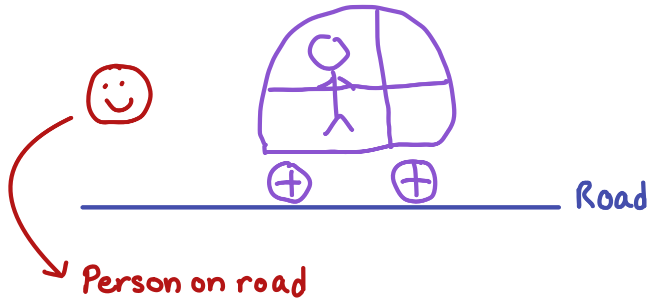
Coriolis Effect – A fictious force that acts on objects which are in motion in a rotating FOR w.r.t an inertial FOR.

Newton's laws describe objects in an inertial (non-accelerating) FOR. When Newton's laws are transformed in a rotating FOR, Coriolis & Centrifugal accleration appear.

Note: A FOR consists of an abstract coordinate system and the set of physical points that uniquely fix (locate & orient) the coordinate system and standardise measurement within that frame.

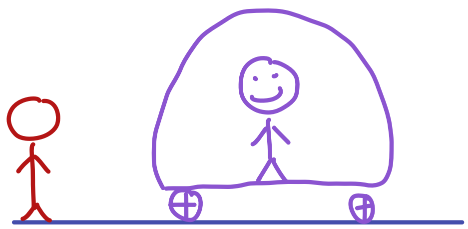
Example.

1)



To an observer in an inertial FOR (The road) the car will seem to acclerate. (He sees the car moving). In order for the passenger to remain in the car, a force must be exerted on the passenger. (A force that makes the passenger move with the car). The seat forming part of the car is also acclerating and is thus exerting the force that moves the passenger together with the car. Remember, the passenger is not moving himself with the car, it is the latter which is moving the passenger. This causes the passenger to acclerate in the FOR of the road.

2)



From the P.O.V of the interior of the car, an acclerating FOR, there is a fictious force pushing the passenger backwords. (This force balances him with the car making him feel he is stationary (const. Velocity) despite being in an acc. FOR) This force pushes the passenger back in the seat until the seat compresses and provides an equal and opposite force. The passenger is stationary in this frame because the fictious & real forces balance.

The passenger is actually moving forward with the car. Although he is in a state of accleration, he does not feel/see a forward force as he is moving together with the car.

He also knows that he can't move out of the car although he too is acclerating. There must therefore be a force to block his accleration, but not that of the car. This force should be provided by the car to "kill" the feeling of going forward (which is not experienced by people in car as going same rate as car) & to go in accord with nature's laws (Newton). This backword force (experienced) is the fictious force!

Circular Motion.

→ Fictious forces make the contents of a non-inertial FOR stationary so that Newton's laws can still apply.

↳ Property of Nature : Everyone should be able to experience Newton's laws. (Even if they are rotating/acc.)

↳ Stationary outside observers need not this effect to apply Newton's law.

Everyone has his own FOR.

- From the viewpoint of an inertial FOR (w.r.t the road) [Stationary] a car at a roundabout is accelerating towards the centre of a circle (Centripetal force provided by friction between wheels & road). The car is accelerating due to unbalanced force. [Nothing to counter friction between wheels & road].

- From the viewpoint of a rotating Frame, moving with the car, the fictious centrifugal force tends to push the car outside of road & occupants outside of car. The centrifugal force balances the friction making the car stationary in the non-inertial FOR.

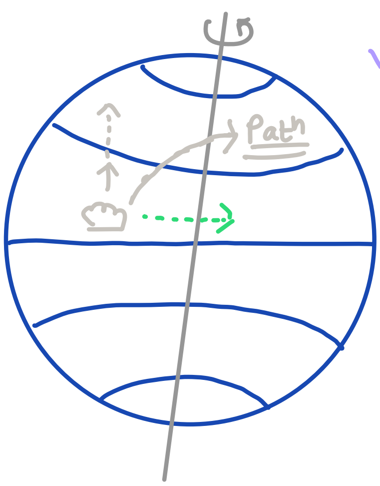
FICTIOUS FORCES ARE PART OF NON-INERTIAL FOR, TO MAKE NEWTON'S LAWS APPLY IN THESE TOO. USUALLY, THEY WORK ONLY IN INERTIAL FOR, BUT FICTIOUS FORCES MAKE NLs WORK FOR EVERYONE!

NLs work only in Inertial FORs.

WHY??

Coriolis Force. → Deflected to right w.r.t direction of travel in Northen hemisphere and to left in southen hemisphere.

Coriolis Effect in Meteorology



$v = \omega r \Rightarrow v \propto r$

$2\pi r$

$\Rightarrow$ Objects at equator rotate faster (Cover greater distance in same time)

$F = \frac{mv^2}{r} = \frac{mv\omega r}{r} = mv\omega \Rightarrow F \propto v \propto r$

- Suppose a cloud is moving towards N-pole. Remember that due to Earth, it is also moving towards the EAST. As North drawing force becomes stronger closer to the poles, its influence becomes greater $\Rightarrow$ Curved Trajectory

Suppose a cloud is now moving south. It is still moving EAST due to

EARTH. The South-drawing forces become stronger also causing a curve.

Gravitational Force provides Centripetal accleration.

$\Rightarrow F_c = mg$

Remember Reaction is directed opposite to weight

$\Rightarrow F_c = mg - F_R$

At pole $F_c = 0$

$\Rightarrow F_R = mg$ (real weight)

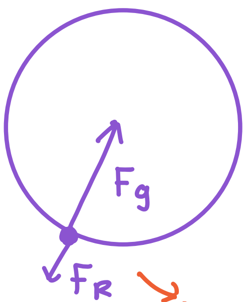
At equator, object performs Circular motion.

F_c at equator $= m\omega^2 r$

$F_c = mg - F_R$

$F_R = mg - \underbrace{F_c}_{\text{Non-zero}}$

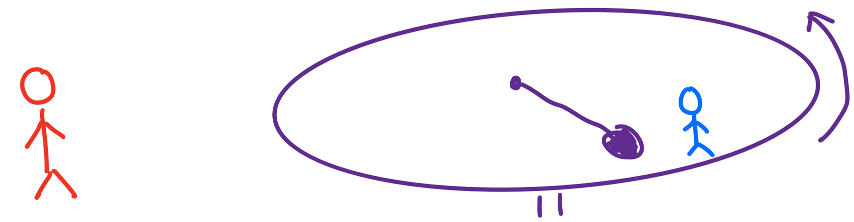
$\Rightarrow$ Weight Equator < Weight Pole.



F_R is the force provided by Earth to prevent us from falling towards its centre!

The net gravitational force which results provides the centripetal force! If earth rotate fast enough, s.t $F_c > mg$, F_R becomes -ve making everything bury in the ground. ??

Coriolis Effect



Suppose a man is standing on the ground and looking at the rotating table.

He will see a body rotating around the nail with a Tension = $F_c = \frac{mV^2}{r}$

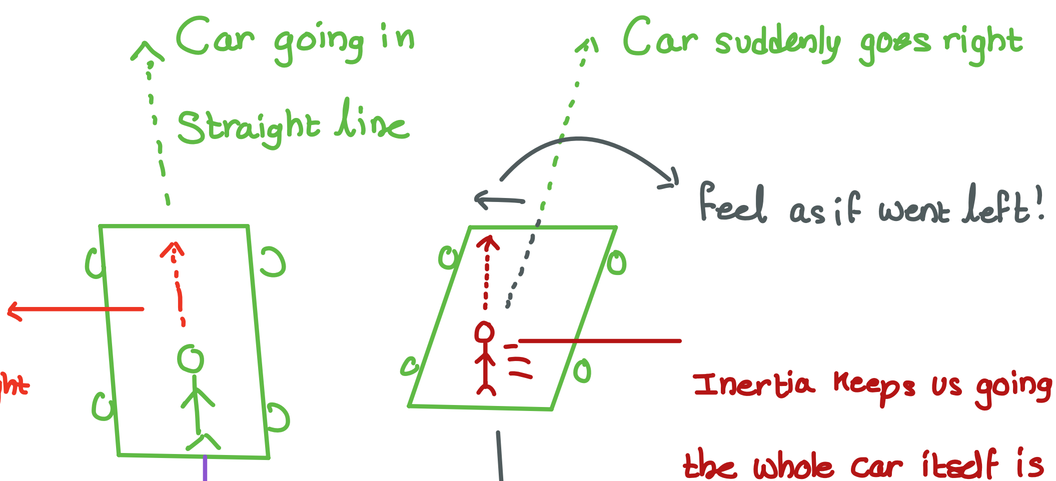
For the man standing on top of the table, the body will appear at rest because the body's position is not changing w.r.t the man. Thus from the p.o.v of the man, there should be no force acting on the body. But he knows a force (centripetal) is acting inward on the body and that an outward force of same magnitude must be present to keep the body at rest. [The outside man knows this is fake as he sees the body rotating. But for the man on the table, he cannot reason that there is a centripetal force which he knows plus the body being at rest. He thus concludes that there must be a radially outward force to keep the body at rest!]

But what provides the centrifugal force and why is it felt if it is fictitious?

↳ This "fake force" is felt due to a change in momentum (Our Inertia)

Suppose we are in a car going in a straight line and then it suddenly turns RIGHT. Why do we veer left instead of right - direction of the centripetal force?

We are going in straight line too!



Inertia keeps us going straight. Since the whole car itself is going right while

We do not notice that we are going forward as we are not changing posn w.r.t Car. In this FOR we think we are in equilibrium with Car.

Both this are forces

as mass is acclerating "relatively"

Mass acclerating in Fixed coordinate system ⇒ Real force.

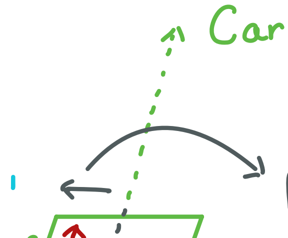
Both going forward no care only lateral is acclerating

Mass "fixed" in acclerating coords. System ⇒ Fictious force.

Since as car rotates forward directin also changes slightly, we experinced a "curved" force.

Car suddenly goes right

Actually like this!



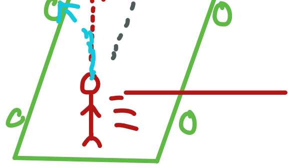
Feel as if went left!

we are not, we see as if we are going left. - Really inertia cause us to lack behind the changing motion of the car.

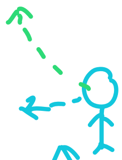
We inside our car think that everything is normal (We cannot know that the car is in fact acclerating - $F_{centripetal}$)

To explain the phenomena we felt, (Veering left, we call it the fictitious force

This force infact does not exist! The real reason was the Centripetal force acting on the whole FOR. Since we cannot know that our whole FOR was being acted upon, We call the left - veering effect of $F_{centripetal}$ the $F_{centrifugal}$. The latter is only a combination of our inertia and a changing FOR. Since the mass is fixed and Coordinate System is changing, a force also occurs.



Change in direction cause $V_{forward}$ to decrease also slightly



Hit left window directly in theory

Go a little forward & hit left actually.

This occurs because as the car turns his forward velocity is now slightly lower – A "forward deceleration"
The person does not know that the car/his FOR has descelerated and thus feel a little forward accleration (by symmetry). This gives the forward curved path!

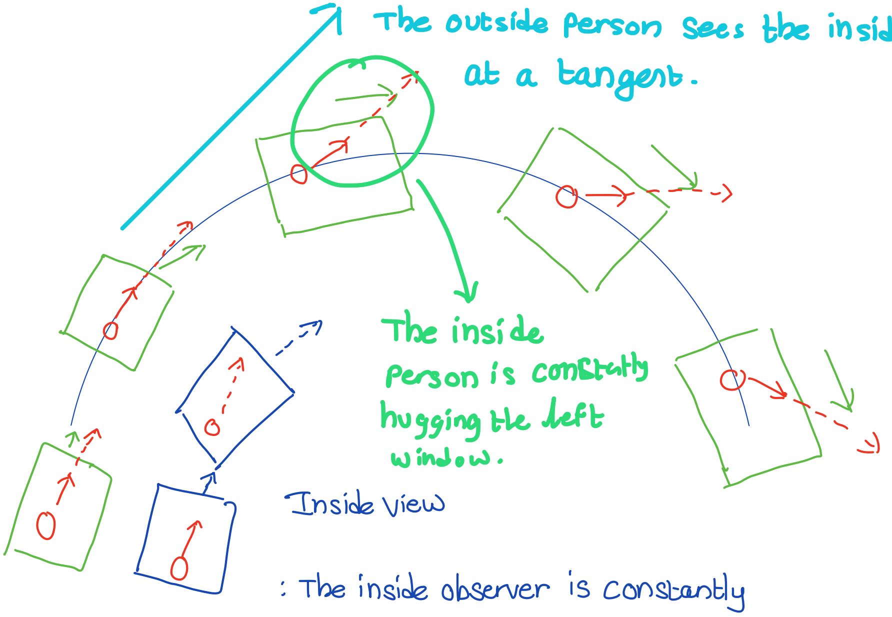
As the car turns right the person is still going forward Giving the impression that he went left.

What would the person outside see?

Drawing of what is actually happening.

Outsider will see the person inside "missing" hitting the window at each time interval!

Direction of Car

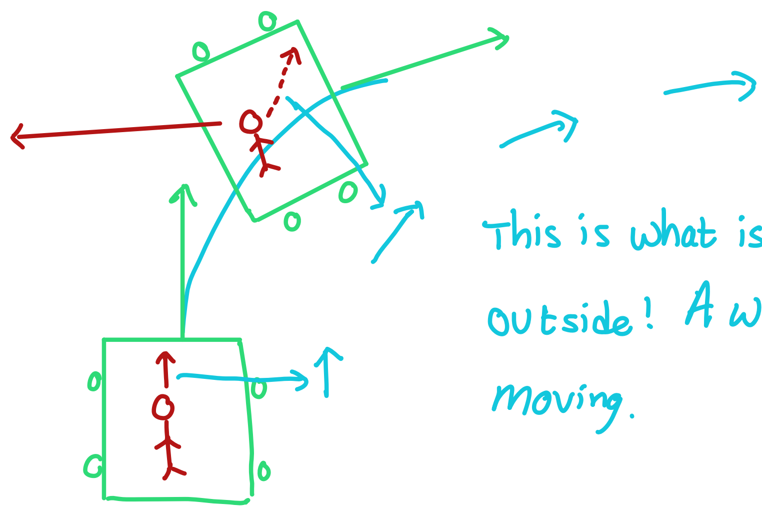


Outside View

: The inside observer is constantly hitting the left wall due to this centrifugal force.

The observer outside will never see the person hitting the wall but as if "always miss it" making him appear "one" with the car. He sees that as the person is about to hit the wall, the car already moved to prevent hitting the wall. Since the other person is moving with the car, the effect of his inertia cause him to appear to veer left while infact it is the car which is moving right while his inertia makes him maintain previous motion/stay still.

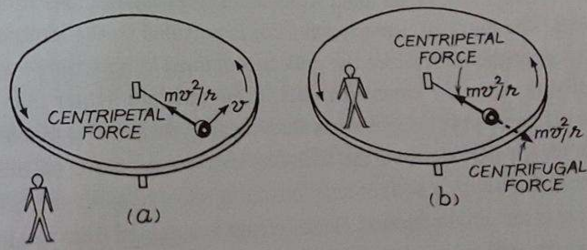
The object will hit the left wall from the inside view



This is what is seen from the outside! A whole block (Person + car) moving.

6. Centrifugal Force : [A Pseudo Force:

There are certain situations in which we feel that a body is acted upon by a force, but actually there is no force on the body. Such an apparent force is called a 'pseudo force'. Suppose that a body of mass m is placed on a smooth circular table which is rotating with a uniform angular velocity around a nail passing through the centre of the table. Suppose the distance of the body from the nail is r and the linear velocity of the table at the body is v. If there is no sufficient friction between the table and the body, then in order to keep the body at rest on the table it must be tied to the nail by a string.

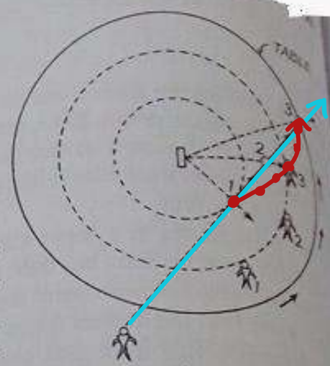


(Fig. 14)

A man standing on earth and looking at the rotating table will see the body rotating around the nail and find on it a centripetal force $m\,v^2/r$ (Fig. 14 a). This force is provided by the tension in the string and is a real force acting on the body.

To another man, however, who is standing on the table, the body appears at rest because its position is not changing with respect to the man (Fig. 14 b). Thus, from the point of view of this man,

there should be no force acting on the body. But actually the body is acted upon by an inward force $m\,v^2/r$. Hence according to the man standing on the moving table, a force of magnitude mv^2/r is also acting outwards so that the net force on the body is zero. This apparent outward force is called 'centrifugal force'. Although the centrifugal force is an apparent force, but its effect is clearly observed by the man standing on the table. If the string of the body is cut then to the man standing on the table, the body would appear moving radially outwards (Fig. 15). According to him, the centripetal force being no more (as the string has been cut), the body is moving outwards under the centrifugal force.* If, however, the same event is observed by the man standing on earth, then according to him, as soon as the string is cut, the body moves in the direction of the tangent to the circular path, as required by Newton's first law. This man would not feel any centrifugal force.



(Fig. 15)

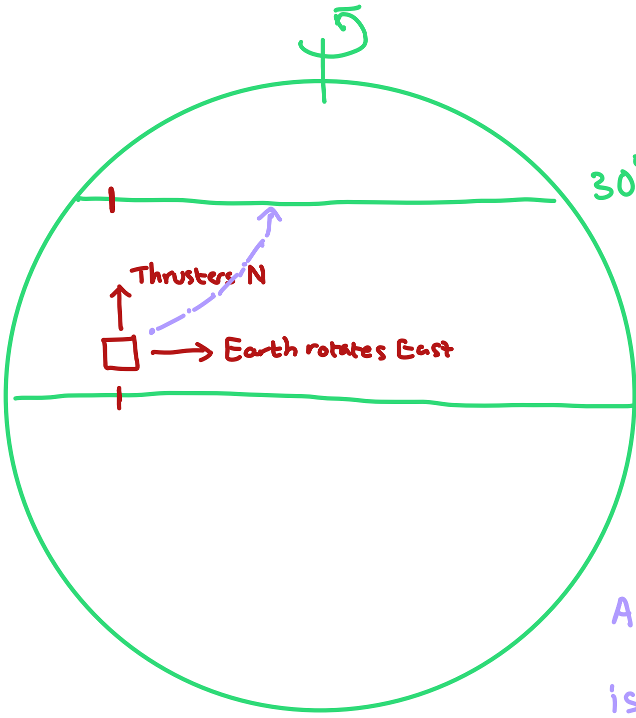
— Man on wheel
— outside man

Infact, when a person stands on a rotating platform then he feels the centrifugal force and its effect. If we stand on a rotating merry-go-round without holding its strings, we would fall outwards as if we have been acted upon by some (centrifugal) force. To stand on the merry-go-round we must stretch the strings outwards so that by action-reaction law, the string may pull our body inwards and provide the necessary centripetal force. Similarly, if we are sitting in a train which suddenly turns towards right, then we are struck with the left-hand wall of the train. This is not because some real force directed towards left has acted upon us. Infact, when the train turned to the right, our body continued to move straight and we struck with the left-hand wall and felt a (centrifugal) force acting on our body.

Centrifuge : It is a device to separate the particles of different masses present in a liquid. In this device the liquid is filled in a vessel, or in tubes, and rotated at a very high speed. In this process the heavier particles adopting a path of larger radius, move towards the circumference and are separated. The cream is separated from the milk by this method.

Suppose a vessel filled with milk is rotating. The milk rotates along with the vessel about the same axis. The necessary centripetal force is provided by the internal frictional force (viscosity) between the layers of the milk. Since the milk is contained in a rotating vessel, its particles experience a centrifugal force $m\,v^2/r$. The cream particles in the milk are lighter than the milk particles. The lighter particles would experience a lesser force than the heavier particles moving along a path of the same radius. Hence the cream particles and the milk particles cannot move on the same path. The lighter cream particles tending to adopt a path of smaller radius move towards the centre and the heavier milk particles move towards the circumference. Hence cream is obtained from the inner part of the vessel and the milk from the outer part.

Earth



30° Rotates relatively slower $\frac{(2\pi r_1)}{t}$

0° Rotates relatively faster $\frac{(2\pi r_0)}{t}$

For FOR Earth: *

As the plane flies N, the Earth is also rotating but we do Not know so. We are on Earth.

A person Far in Space see the plane Going NE in a straight line as the rotation of the earth nullifies the appearance of the package curving.

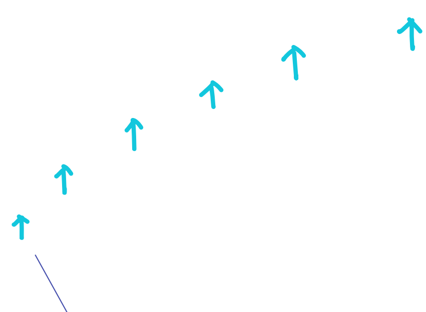
↳ As the package is about to curve, Earth countries (coords./location) already catch up with it making it seem that it travels in straight line NE.

↘ East travel occurs due to Earth rotation.

* On Earth, we notice the curvature which is infact our planet rotating and us not seeing it which makes it appear as if we are going East.

↓ FOR.

Like this



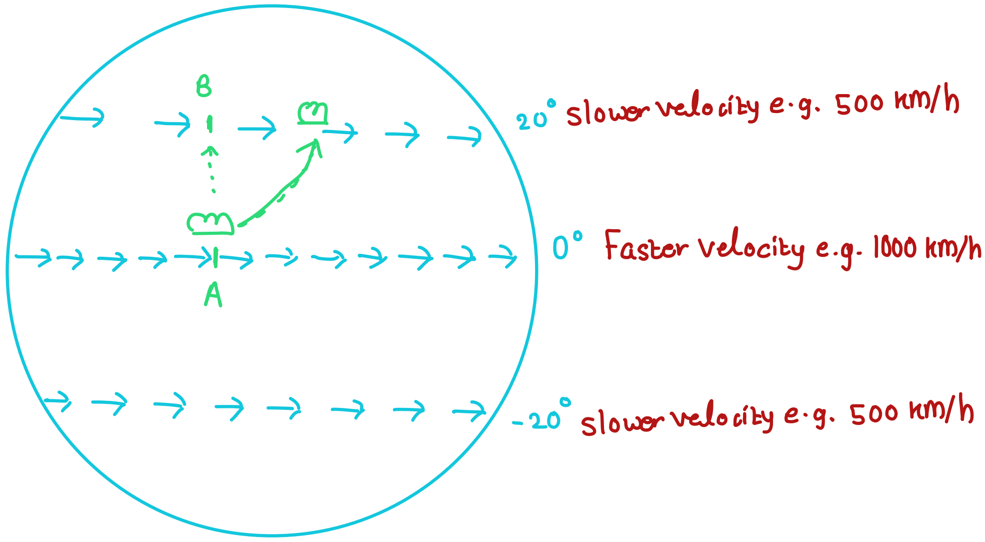
This occurs as as we go to higher latitudes, land travels Slower → From release at equator land at 30° has not yet travelled same distance.

We still have our thrusters blasting North!

But remember we are moving together with the Earth

↳ When viewed from far we are moving NE

$V = \omega r$



→ Land on 20° can move slower than at 0° since it has a smaller arc to cover to remain "in line"/over A.

Suppose a cloud is present at A (0°). It is travelling with an angular velocity of 1000 km/h Eastwards. Suppose a gust of wind pushes it northwards. It retains its angular velocity of 1000 km/h due to angular momentum (Inertia - No force exist to counter eastward velocity) as it travels North. When it arrives at 20° during its travel it retaining its equatorial velocity moved East "more" compared to the land there. (a drift). We see the drift as a deflection towards the right. [The only way to stop this effect is to stop Earth from rotating or constantly rotate c/w - cc/w shifting quickly periodically]. Trying to apply a Force to the left to stop the curving is futile - the Cloud would still curve - However, when properly calculated, doing so would make the cloud land in the right spot along a still curved path. The curving is due to the immutable cc/w rotation of the Earth.

A B

Now consider an object moving from -20° to 0° Northwards. At -20° it has and retains a lower angular velocity compared to the equator. When the cloud arrives at 0° the Equatorial initial location of choice (B) having higher angular velocity has already moved by giving the impression that the cloud has curved left!

The Curving originates from the Cloud being not connected to Earth (like on land). Thus when it moves N or S, it retains its initial angular velocity which does not change in relation to different latitudes - Why??

↳ When set in motion, freely moving objects move in straight paths while Earth continue to rotate independently. These objects are not carried with Earth as it rotates.

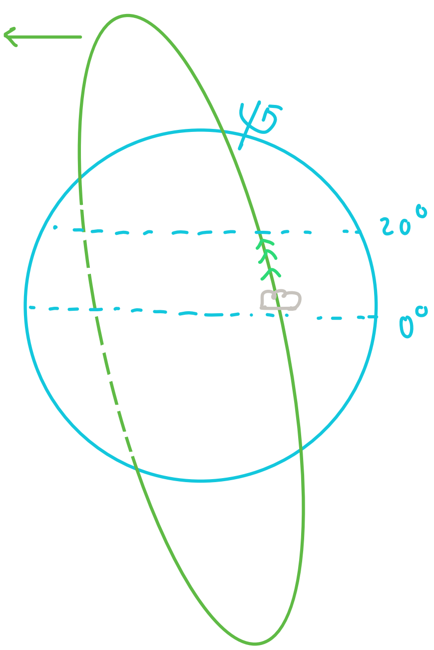
↳ But if winds/clouds/planes are not connected to Earth, why do possess the initial angular velocity of their latitudes? ⇇ Question!!

Now consider moving South wards from $\overset{A}{20^\circ}$ to $\overset{B}{0^\circ}$. At 20°, Object has lower a.v. When it reaches B, the latter already moved by giving the impression of a left-deflection.

From $\overset{A}{0^\circ}$ to $\overset{B}{-20^\circ}$ → Object is faster than at -20°. Moving south, it had higher a.v than B which makes it ahead giving impression of a right deflection.

When viewed from an Inertial FOR, the person see the clouds moving in a straight line independant of Earth's rotation! - Actually the arc of a great circle

Imaginery great circle representing Atmosphere. ←



He sees the cloud moving along that path "disembodied" from Earth.

↳ That path is a straight line - or the arc of a great circle

↑ Inertial FOR.

On Earth, We are connected to land and the Earth is our FOR (a non-inertial one). It is in

Objects moving East have increased ω in addition to Earth's rotation $\Rightarrow F_{centrifugal}$ also increases. $\Rightarrow$ reduction in F_{grav}

this which we percieve the curving or coriolis effect! As we are not "disembodied"/disconnected from Earth but rotating/observing on it!

Perceived Change in grav force due to change in $F_{centrifugal}$

Note: Eötvös Effect - Moving East to West. (Not shown)

Coriolis in East-west

$F_c = \frac{mv^2}{r}$ $\rightarrow$ g provides F_c

$\Rightarrow g = \frac{mv^2}{r}$ but g is a constant

$\Rightarrow$ to maintain g

At small $r \rightarrow V$ must decrease

At large $V \rightarrow r$ must increase too!

East/west coriolis

$\rightarrow$ go fast $\Rightarrow$ fly out / go slow $\Rightarrow$ fall in (than "allowed")

Objects moving East will want to "fly out" while west will want to dive in.

$F_{NET} = F_{Grav} + "F_{coriolis}" + "\cancel{F_{centrifugal}}"$

$F_{NET} \downarrow K$

Ignore for now

Move East + Eastern Earth rotation makes this bigger.

To keep F_{NET} constant

F_{grav} which is really $F_{centripetal}$ must decrease $F_c = \frac{mv^2}{r}$

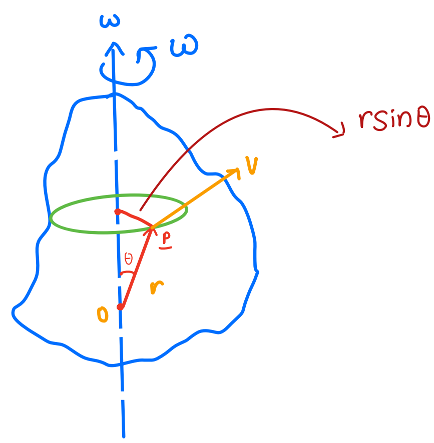
V is constant for a given latitude

$\Rightarrow$ to make $F_{centripetal}$ smaller, r must increase akin to flying out!

But Why Curving (In Coriolis Effect)?!

① Angular Velocity (in 3D)

$$\dot{r} = \omega \times \hat{r}$$



A rigid body rotates about an axis through point O with angular speed ω. Prove that the linear velocity of point $\underline{P}$ with p.v r is given by $V = \omega \times r$

$\underline{P}$ travels in a radius $r\sin\theta$.

defn of cross product.

The magnitude of this linear velocity is $|(\omega \times r)| = \omega r \sin\theta$

$\Rightarrow V = \omega \times r$ / $\dot{r} = \omega \times r$

r, ω, V

↳ $r\sin\theta \times \omega$ (both mutually ⊥) gives V

② Transformation between Inertial and rotating frames.

Consider an arbitrary vector $\bar{a}$ in the rotating body.

$$\bar{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \quad (\text{Pos}\underline{\text{n}})$$

In Newtonian mechanics, Scalar quantities must be invariant for any given choice of frame.

$$\Rightarrow \quad \left.\frac{da_x}{dt}\right|_{\underset{\text{Inertial}}{I}} = \left.\frac{da_x}{dt}\right|_{\underset{\text{Rotational.}}{R}}$$

where $\frac{da_x}{dt}$ represents speed and not velocity.

Same for a_y & a_z.

Hence any transformation of $\bar{a}$ between frames results from changing unit vectors of the basis. (Coord. change) – All magnitudes remain the same – It is the coordinates which are shifting.

$\dot{r} = \omega \times r \qquad \dot{r} = \frac{d\bar{r}}{dt}$

Product Rule! $\rightarrow \left.\frac{d\bar{a}}{dt}\right|_I = \frac{d}{dt}\left(a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}\right)$

$$= \left(\frac{da_x}{dt}\hat{\imath} + \frac{da_y}{dt}\hat{\jmath} + \frac{da_z}{dt}\hat{k}\right) + \left(a_x\frac{d\hat{\imath}}{dt} + a_y\frac{d\hat{\jmath}}{dt} + a_z\frac{d\hat{k}}{dt}\right)$$

$$= \left.\frac{da}{dt}\right|_R + \left(a_x\omega \times \hat{\imath} + a_y\omega \times \hat{\jmath} + a_z\omega \times \hat{k}\right)$$

$$= \left.\frac{da}{dt}\right|_R + \omega \times \bar{a}$$

As shown

Now consider a p.v on the surface of a body $\Rightarrow$ we can write

$$V_I = \left.\frac{dr}{dt}\right|_I = \left.\frac{dr}{dt}\right|_R + \omega \times r$$

Similarly for $\underline{a} = V_I$

$$\left.\frac{d^2r}{dt^2}\right|_I = \left(\left.\frac{d}{dt}\right|_R + \omega \times\right)^2 r = \left.\frac{d^2r}{dt^2}\right|_R + 2\omega + \left.\frac{dr}{dt}\right|_R + \omega \times (\omega \times r)$$

Forces on body in a rotating Frame.

Consider a force (e.g. Gravity) acting on an object as posn r. N2L states

$$F = m\left.\frac{d^2r}{dt^2}\right|_I$$

$\rightarrow$ Subs this into previous eqn for $\left.\frac{d^2r}{dt^2}\right|_I$ and rearranging

we get:

$$F_{NET} = m\left.\frac{d^2r}{dt^2}\right|_R$$

$$= F - 2m\omega \times V_R - m\omega \times (\omega \times r)$$

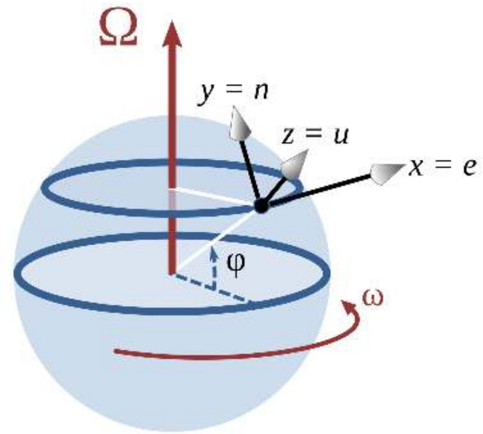
$$= F - \underbrace{2m\omega \times V_R}_{\text{Coriolis force.}} + \underbrace{m\omega^2 r}_{\text{Centrifugal force.}}$$

Coriolis force.

$-2\omega \times V_R$

$\rightarrow$ Coriolis acc.

Centrifugal force. (Pointing away from Centre of rotation.)



Rotating sphere.

Consider a location with a latitude ϕ on a sphere that is rotating around the North-South axis. A local coordinate system is setup with the x-axis due East, the y-axis horizontally due North and the z-axis vertically upwards. The rotation vector, velocity of movement and Coriolis accleration expessed in this local coord. System (e) East, (n) North and upward (u) are :

$$\Omega = \omega \begin{pmatrix} 0 \\ \cos\varphi \\ \sin\varphi \end{pmatrix}, \quad v = \begin{pmatrix} v_e \\ v_n \\ v_u \end{pmatrix}$$

$$a_c = -2\,\Omega \times v$$

when considering atmospheric or oceanic dynamics, the vertical velocity is small and the vertical component of the Coriolis accleration is small compared to the accleration due to gravity. For such cases, only the horizontal (east and north) components matter. The restriction above the horizontal plane is setting $v_u = 0$:

$$v = \begin{pmatrix} v_e \\ v_n \end{pmatrix}, \quad a_c = \begin{pmatrix} v_n \\ -v_e \end{pmatrix} f$$

latitude

Coriolis Parameter $= 2\omega\sin\varphi$

By setting $v_n = 0$, it can be seen immediately that (for positive ϕ and ω) a movement due East results in an accleration due South. Similarly setting $v_e = 0$, it is seen that a movement due North results in an accleration due East. In general, observed horizontally, looking along the direction of movement causing the accleration, the accleration is turned 90° to the right and of the same size regardless of horizontal orientation.

As a different Case, consider Equatorial motion setting $\phi = 0$. In this case, Ω is parallel to the north axis, and :

$$\Omega = \omega \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} v_e \\ v_n \\ v_u \end{pmatrix}, \quad a_c = -2\Omega \times v = 2\omega \begin{pmatrix} -v_u \\ 0 \\ v_e \end{pmatrix}$$

Accordingly, an Eastward motion (that is, in the same direction as the rotation of the sphere) provides an upward accleration known as the Eötvös effect, and upward motion produces accleration due west.

Coriolis Parameter

$f = 2\Omega \sin\varphi$ ← latitude

rotation rate of Earth

$f = 10^{-4}$ rads^{-1} in midlatitudes → Frequency of Inertial oscillations on Surface of Earth. ← result of coriolis Effect.

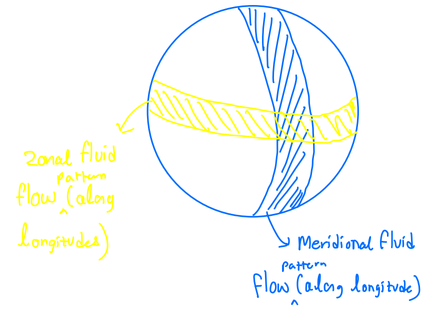
$\Omega = \frac{2\pi}{T} = 7.2921 \times 10^{-5}$ rads^{-1} $T = 1$ sidereal day (23h 56min 4.1s)

↘ like Absolute magnitude but for time.

Explanation.

Consider a body (e.g. fixed volume of atmosphere) moving along a given latitude φ at velocity V in Earth's rotating FOR. In the local FOR of the body the vertical direction is parallel to the radial vector pointing from the centre of the Earth to the location of the body and the horizontal direction is perpendicular to this vertical direction and in the meridional direction. The coriolis force (proportional to $(2\Omega \times V)$), however is $\perp$ to the plane containing both the Earth's angular velocity $\bar{\Omega}$ (Where $\Omega = |\bar{\Omega}|$) and the body's own velocity in the rotating FOR V. Thus, the coriolis force is always at an angle φ with the local vertical direction. The local horizontal direction of the Coriolis force is thus $\Omega \sin\varphi$. This force acts to move the body along longitudes or in the meridional direction.

Equilibrium.



Suppose a body is moving with velocity v s.t the $F_{centripetal}$ and $F_{coriolis}$ (due to Ω) on it are balanced.

We then have

$$v^2/r = 2(\Omega \sin\varphi)v$$

r ↘ radius of curvature of path of object (defined by v). Replacing $v = \omega r$

Ω ↑ magnitude of spin rate of Earth

We get $f = \omega = 2\Omega \sin\varphi$

(or Coriolis frequency)

$\Rightarrow$ Coriolis Parameter f is the angular velocity/frequency required to maintain a body at a fixed circle of latitude or zonal region. If f is large, the effect of Earth's rotation on the body is significant since it will need a larger angular velocity to stay in equilibrium with the Coriolis Effect. Alternatively, if f is small, the effect of Earth's rotation is small since only a small fraction of the centripetal force on the body is cancelled by the Coriolis force. Thus the magnitude of f strongly affects the relevant dynamics contributing to the body's motion. These considerations are captured by the Rossby number

$$\beta = \frac{\partial f}{\partial y}$$

→ rate of change of f along the meridional direction is significant in stability calculations.

↓ Important in Rossby waves.

y : local direction of increasing meridian.

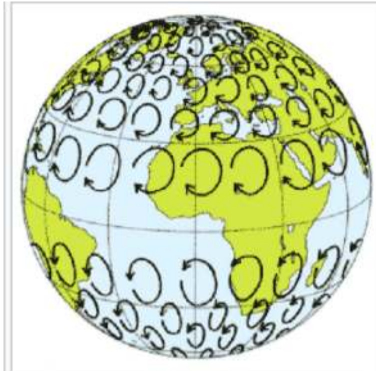
Inertial Period / Inertial circles.

- Regardless of their speed, freely moving objects at the same latitude appear to complete a circle and return to their original location in the same period of time (called inertial period) - 12 h at poles increasing at lower latitudes. 24 h 30°N or 30°S - Infinity near the equator.

CC.12 The Coriolis Effect

ESSENTIAL TO KNOW

- When set in motion, freely moving objects, including air and water masses, move in straight paths while the Earth continues to rotate independently.
- Because freely moving objects are not carried with the Earth as it rotates, they are subject to an apparent deflection called the "Coriolis effect." To an observer rotating with the Earth, freely moving objects that travel in a straight line appear to travel in a curved path on the Earth.
- The Coriolis effect causes an apparent deflection of freely moving objects to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. The deflection is said to be cum sole, or "with the sun."
- The Coriolis deflection is greatest at the poles and decreases at lower latitudes. There is no Coriolis effect for objects that move directly east to west or west to east at the equator.
- Regardless of their speed, freely moving objects at the same latitude appear to complete a circle and return to their original location in the same period of time. This period, called the "inertial period," is 12 h at the poles and increases progressively at lower latitudes. The inertial period is 24 h at 30°N or 30°S, and it approaches infinity near the equator.
- For objects moving within 5° on either side of the equator, the Coriolis effect often can be ignored because the deflection is small and the inertial period is very long.
- Freely moving objects moving at the same speed appear to follow circular paths with smaller radii at higher latitudes.
- Freely moving objects at the same latitude appear to follow paths with larger radii if they are moving at higher speeds.
- The magnitude of the Coriolis deflection (the rate of increase of distance from a straight-line path) increases with increasing speed.



Schematic representation of inertial circles of air masses in the absence of other forces, calculated for a wind speed of approximately 50 to 70 m/s (110 to 160 mph).

Inertial circles [edit]

An air or water mass moving with speed v subject only to the Coriolis force travels in a circular trajectory called an 'inertial circle'. Since the force is directed at right angles to the motion of the particle, it moves with a constant speed around a circle whose radius R is given by:

$$R = \frac{v}{f}$$

where f is the Coriolis parameter $2\Omega \sin \varphi$, introduced above (where φ is the latitude). The time taken for the mass to complete a full circle is therefore $2\pi/f$. The Coriolis parameter typically has a mid-latitude value of about 10^{-4} s^{-1}; hence for a typical atmospheric speed of 10 m/s (22 mph) the radius is 100 km (62 mi), with a period of about 17 hours. For an ocean current with a typical speed of 10 cm/s (0.22 mph), the radius of an inertial circle is 1 km (0.6 mi). These inertial circles are clockwise in the Northern Hemisphere (where trajectories are bent to the right) and anticlockwise in the Southern Hemisphere.

If the rotating system is a parabolic turntable, then f is constant and the trajectories are exact circles. On a rotating planet, f varies with latitude and the paths of particles do not form exact circles. Since the parameter f varies as the sine of the latitude, the radius of the oscillations associated with a given speed are smallest at the poles (latitude = ±90°), and increase toward the equator.[42]

Cloud formations in a famous image of Earth from Apollo 17, makes similar circulation directly visible

Other terrestrial effects [edit]

The Coriolis effect strongly affects the large-scale oceanic and atmospheric circulation, leading to the formation of robust features like jet streams and western boundary currents. Such features are in geostrophic balance, meaning that the Coriolis and *pressure gradient* forces balance each other. Coriolis acceleration is also responsible for the propagation of many types of waves in the ocean and atmosphere, including Rossby waves and Kelvin waves. It is also instrumental in the so-called Ekman dynamics in the ocean, and in the establishment of the large-scale ocean flow pattern called the Sverdrup balance.