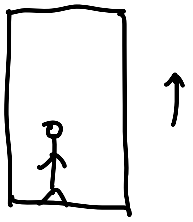
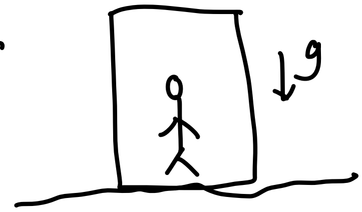
Principle of Equivalence. ①

Suppose we are in a box



We cannot tell if we are in outerspace acc. up $g \text{ ms}^{-2}$

or

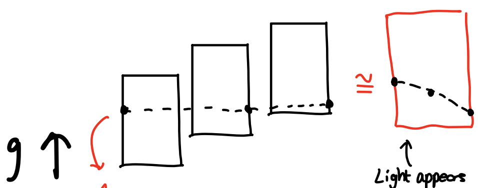


On Earth stationary at surface (with acc. $g \text{ms}^{-2}$ down)

No Experiment can distinguish between these 2.

But In outer space, all points of the box are accelerating $g \text{ m s}^{-2}$ BUT on Earth gravity varies with height g at bottom is slightly greater than at top. → That difference is the TIDAL FORCE.

② Light bends in a grav. field.



light moving in straight line but as the box moves it seems that it goes lower down

As we shine the light from the left, the box is also moving up. As the light reaches RHS,

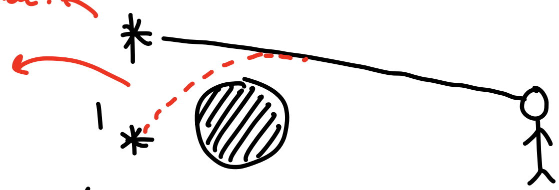
But since equivalence principle says this is same for Earth, → The same thing should happen on Earth i.e. Light bends in a grav. field.

↓

Shown to be true during Solar Eclipse.

Give impression Star here.

light from star bent in Sun grav. field



moon blocks the sun

Could see star slightly right of sun

But Astronomers knew Star was located behind the sun.

Shouldn't have been possible to see.

Problem for Einstein - Newton Grav Law

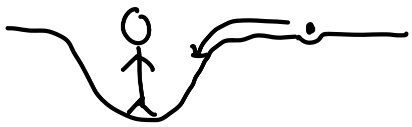
$$F = \frac{GMm}{r^2}$$

Light was bending in a field but photons have no mass!

Instead of gravity

Einstein Propose: All forms of motion are represented by movement in curved space time.

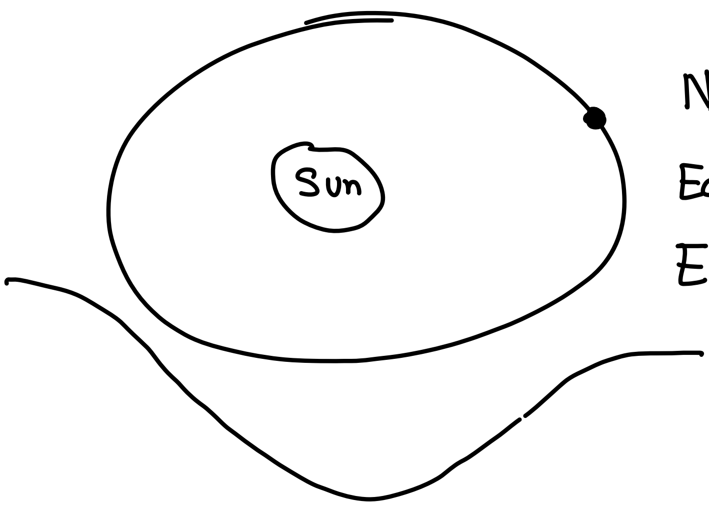
small marble on trampoline cause small dent



Movement of marble towards larger pit is [the shortest path in curved Space time]

↳ Einstein

Newton Said - Movement towards person is caused by grav. Attraction.

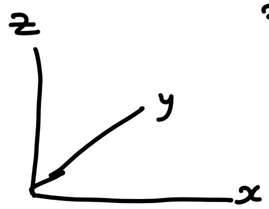


Newton: Grav. force provides $F_{centripetal}$ to keep Earth in Orbit.

Einstein: Massive Sun curves space time & Earth goes round that Curve in straight line in Space time.

SpaceTime – Space & Time as one set of Coordinate

Space:



3-Orthogonal axes (3D)

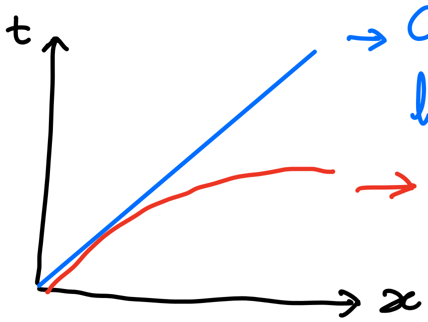
+ Time = 4D Spacetime

↓

Difficult to draw!

✳

✳ Draw 1 Space dimension + time



→ Constant velocity yields straight line. (Cons.dist / Cons.time)

→ Acc. Cause a curve

↳ Since grav. Cause acc

⇒ grav cause curve in spacetime COORDINATE.

Magnificent ←

[Einstein thought : SpaceTime must be curved since acc. has the effect of creating a curve in SpaceTime Coordinate.]

Einstein Field Equations

Stress-Energy-Momentum Tensor.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricci Curvature Tensor

metric tensor

Curvature Scalar

Cosmological Constant

Refers to mass-Energy

↳ Refers to Curvature of space-Time

There are actually many eqns : indices μ & ν represent dimensions of Spacetime : 0 time, 1 x, 2 y, 3 z

Combination of μ & ν ←

⇒ 16 Variations of this eqn

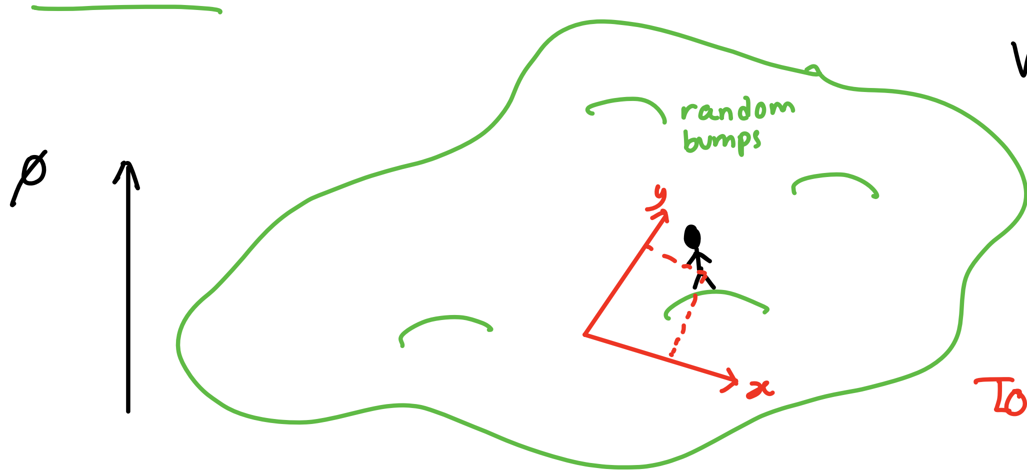
10 eqns ← But 6 are duplicates

Mass tell space time how to curve & Curved space-time tells mass how to move.

Metric Tensor

Consider any field : Temperature, electromagnet, gravitational, ...

↘ Land field



Value of field ϕ is height above Sea level.

- Since field is bumpy, ϕ will constantly change

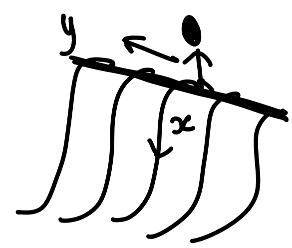
To locate ourselves, we impose a set of coordinates

- Uniquely define posn and for every posn I take I can find a value of ϕ → Height above sea level of point (x, y) → In the field

How does the height change if I move a little bit.

↳ Depend which part I move.

↳ Suppose we are at the top of a ridge. How can I tell by how much ϕ (height) will vary as I move along?



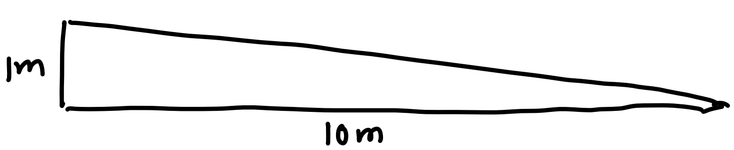
Drop considerably

x - down ridge

y – Along ridge (flat)

↳ Stay level ground.

Consider a $\underline{\underline{1 : 10}}$ gradient → Each 10 m I walk along I drop 1 m down



$$\Rightarrow d\phi = \frac{d\phi}{dx} dx$$

Change in height = Gradient × distance walked along.

$d\phi = \frac{1}{10} \cdot 5 = \frac{1}{2} m$ → If I walk 5 m along a 1:10 gradient, my height changes by ½ m

Change in value of field = Gradient × distance travelled.

↳ Note: Usually field is bumpy & no clear gradient is obtained.
But for very small distances ($d\phi$, dx) this is true!

But direction matters

$d\phi_x = \frac{d\phi}{dx} dx$

↑ change in height while walking in x direction

$d\phi_y = \frac{d\phi}{dy} dy$

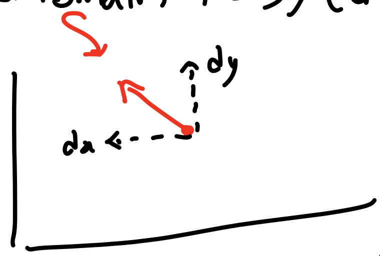
↑ gradient in y direction

Not the same!

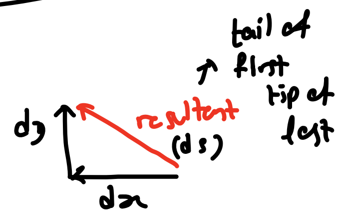
Ridge: x is steep $\frac{d\phi}{dx}$

y is level $\frac{d\phi}{dy}$

Suppose we walk in combination of x, y (diagonal)



Add vectors



Pythagoras: $ds^2 = dx^2 + dy^2$
↑ magnitude

Vector Addition: $d\bar{s} = d\bar{x} + dy$

Subs

$$\boxed{d\phi_s = \frac{d\phi}{dx} \cdot dx + \frac{d\phi}{dy} \cdot dy}$$

↓

$$d\phi_s = \frac{\partial\phi}{\partial x} dx + \frac{\partial\phi}{\partial y} dy$$

↓ Partial derivative w.r.t x only as ϕ could be a function of both x & y.

Separate ← Change in height: * $\boxed{d\phi_s = d\phi_x + d\phi_y}$

from $d\bar{s} = d\bar{x} + d\bar{y}$

Line not derived directly from this?

Along paths

Change in height along path x

for y

↓ These are vectors

Nomenclature change

$x \rightarrow x^1$

$y \rightarrow x^2$

3D : $\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ usual coords. If we need to add other coords. relabelling works better.

time $- x^0$
$x - x^1$
$y - x^2$
$z - x^3$

$$d\phi_s = \frac{\partial \phi}{\partial x^1} dx^1 + \frac{\partial \phi}{\partial x^2} dx^2 + \dots$$

↑ if needed.

↑ can drops at we interpret it as change in field value

$$\boxed{d\phi = \sum_n \frac{\partial \phi}{\partial x^n} dx^n \quad — \quad (1)}$$

$n = 1 \Rightarrow \frac{\partial \phi}{\partial x^1} dx^1$

$n = 2 \Rightarrow \frac{\partial \phi}{\partial x^2} dx^2$

Problem with GR : Any rules we come up with must be independent of FOR.

Special Relativity : Different observers measure different distance & time depending on their relative velocity → Length Contraction & Time dilation.

↳ For something to be universally true, it must be true in all FOR.